Aim for Higher Goals Andree Winner Urges ... p. 3

The Mathematical Log

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COMPETITIONS CAN BE FUN!

School Contest Marks 40th Anniversary

The Mathematical Association of America recently noted the fortieth anniversary of the American High School Mathematics Examination, currently written by almost 400 000 students in over 6000 high schools in the U.S., Canada, and abroad. Designed to stimulate interest in mathematics at high school level, the contest--now in thirty-question, multiple-choice format--was cosponsored by the MAA and the Society of Actuaries. Mu Alpha Theta became a sponsor in 1965, NCTM in 1968, and other groups followed.

Of the approximately 10 000 000 students who participated in the first thirty-nine contests, only 28 have written perfect papers, the MAA report notes.

Focus, the MAA newsletter, invited its readers to try five sample questions from recent examinations. "Remember that no calculators are permitted!" the report directs.

1. Which of the following is closest to $\sqrt{65} - \sqrt{63}$?
   (A) 0.12 (B) 0.13 (C) 0.14 (D) 0.15
   (E) 0.16

2. A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a hole 24 cm across at the top and 8 cm deep. What was the radius of the ball (in centimeters)?
   (A) 8 (B) 12 (C) 13 (D) $8\sqrt{3}$ (E) $6\sqrt{6}$

3. Each integer 1 through 9 is written on a separate slip of paper and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer?
   (A) 0 (B) 1 (C) 8 (D) 9 (E) Each is equally likely.

4. If $\sin(x) = 3\cos(x)$ then what is $\sin(x)\cos(x)$?
   (A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{2}{9}$ (D) $\frac{1}{4}$ (E) $\frac{3}{10}$

5. Suppose that p and q are positive numbers for which
   $$\log_9(p) = \log_{12}(q) = \log_{16}(p + q)$$
   What is the value of $\frac{q}{p}$?
   (A) $\frac{4}{3}$ (B) $\frac{1}{2}(1 + \sqrt{3})$ (C) $\frac{8}{5}$ (D) $\frac{1}{2}(1 + \sqrt{5})$
   (E) $\frac{16}{9}$

Problems, answers, and solutions to all AHSME papers are available in a group of New Mathematics Library publications (to date, vols. 5, 17, 25, and 29).

Answers to the above questions appear elsewhere in this Log issue.

Group 'Power Question' Unique Challenge

Representing a high level contest challenge for a student group at the level of Mu Alpha Theta seniors, the annual ARML Power Question is--reliably--one of the best to come across the Mathematical Log editorial desk. The 1989 question has just arrived from Harry D. Ruderman, an outstanding friend and supporter of Mu Alpha Theta. Dr. Ruderman notes that question preparation is teamwork with colleagues Gil Kessler and Larry Zimmerman, but admits to being "quite proud of the Pythagorean polygon."

The 1989 Power Question follows, with its sample solution, also supplied by Dr. Ruderman, to appear in our Tall Timbers supplement.

Care to get together with mathematically inclined friends and give the Power Question a cooperative try?

* * *

A convex n-gon will be called "Pythagorean" if it has integer sides, it is cyclic, and its longest side is a diameter for its circumscribing circle. It shall be denoted by Pn or Pn:(a,b,...), where a,b,... are the lengths of its sides. We shall always use the letter d for its longest side. [Thus P3 is a Pythagorean triangle. Note that it would be a right triangle.]

I.

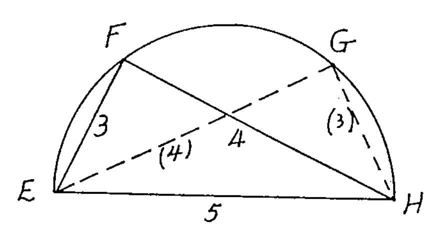
[There is a theorem which states (in part) that: if a prime d is the hypotenuse of a Pythagorean triangle, then d^2 is the hypotenuse of two Pythagorean triangles, d^3 is the hypotenuse of three Pythagorean triangles, etc.]

A. Find two P3's for which d = 25.

B. Find three P3's for which d = 125.

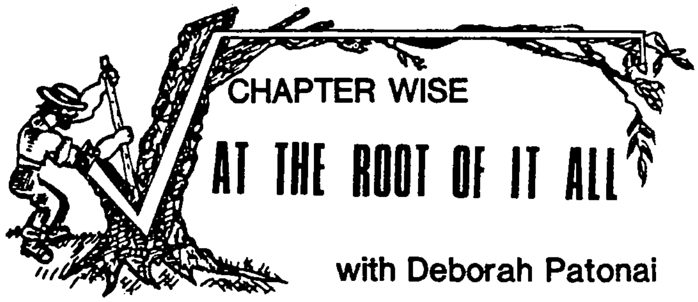
II.

Ptolemy's Theorem says: A convex quadrilateral is cyclic if and only if the product of its diagonals equals the sum of the products of the two pairs of opposite sides.



A. If the P3:(3,4,5) is reflected as above, a quadrilateral EFGH can be formed (it will not be a P4, as FG is not an integer). Multiplying each side by 5 produces a

(See "Group 'Power Question,'" page four)



"Once Mu Alpha Theta, always Mu Alpha Theta," as long-time national secretary-treasurer Harold Huneke has told more than one convention audience, in encouraging students and sponsors to take continuing pride in their affiliation, and, over the years, to keep in touch. When your Editor flew East from Seattle convention in 1987, his seatmate was an engineer, now in management, who had been Mu Alpha Theta in the late fifties ... and remained rightly proud of it. Here, Activities Editor Deborah Patonai tells of a unique effort to maintain a bond which now extends to hundreds of thousands of young Americans.

Ms. Patonai writes:

Have you ever had a "neat" idea ... but, for some reason or other, you've never developed or pursued it? Well, as a Mu Alpha Theta advisor who has attended the last 12 national conventions, I've always thought that inviting all my former Mu Alpha Theta conventioneers together for a "convention reunion" would be interesting and fun. One day I may be able to put my idea into action. However, a Mu Alpha Theta sponsor in Alabama had not only a similar idea, but a more elaborate one. He wanted to hold a reunion of all his former Mu Alpha Theta members. The big difference between this teacher and me is that my Alabama colleague has turned his idea into reality.

Edwin Guthrie, sponsor at West Point High School in Cullman, AL, had a dream of bringing together all former Mu Alpha Theta members in one big reunion. Having served as the chapter's advisor since its inception in 1972, Guthrie had an enormous undertaking ahead of him. Toying with the reunion notion for some time, he finally determined to try it. With the help of current members, he selected December 23 for the reunion, hoping to find as many graduates as possible at home for the holidays. In retrospect, Guthrie's choice was a good one.

Assisting in the formidible task were present Mu Alpha Theta officers Andrea Morton, Christy Quattlebaum, Darlene Kent, Tina Coots, and Joey Skinner, Mu Alpha Theta members, other students, and parents. Invitations and information forms were mailed to about 150 students. From this number, 65 returned their forms and 67 Mu Alpha Theta people attended the reunion.

The present Mu Alpha Theta membership went all out to welcome back their Mu Alpha Theta alumni. They provided a guest register, so that the alumni could "sign in," providing a lasting record. Decorations were everywhere, including poinsettias supplied by a local florist. The members served soft drinks, finger foods, and cakes emblazoned with the Mu Alpha Theta emblem. They also provided everyone with blue napkins with blue lettering, imprinted for the occasion.

The program was carefully planned to work in both current and graduated members. Also in attendance were the school principal and the superintendent. They briefly addressed the gathering, and (we're told) "praised those present for their accomplishments in math competitions tions and for the subsequent successes they have enjoyed." Later, each Mu Alpha Theta introduced himself and made a few brief remarks, if he chose to do so. Ample time was provided for meeting classmates and socializing.

According to Edwin Guthrie, all seemed to enjoy the time spent together. Many suggested that such a reunion be an activity on a regular basis. A considerable number of former students expressed appreciation for the opportunities offered by Mu Alpha Theta affiliation, and indicated that such experiences had been invaluable. Several made reference, we're told, to problem solving skills acquired in competitive events having proven beneficial in college and later as they assumed various roles in society.

This success, to which many alluded, can be linked to their high school days. Many had been active in interscholastic math competitions during their years at West Point High School. In fact, in the fifteen years of Mu Alpha Theta at West Point, students have won approximately

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300 team trophies and awards as well as a greater number of individual awards. Constructing oak trophy cases in numbers suffient to display the trophies, the School proudly featured an exhibit of awards on reunion night.

On sidelight on Mu Alpha Theta graduates at this institution: of 65 information forms returned, 25% of those who responded had earned or were working on degrees in engineering, and another 35% had earned or were working on degrees in other math-related areas. The vast majority of those who replied had continued their formal education beyond high school level.

This reunion of mathematical minds ... a fascinating insight as to how the power of mathematics can charge our lives with energy ... to extraordinary results.

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The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Mu Alpha Theta, founded in 1957 by Richard and Josephine Andree, is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). The Mathematical Log is published quarterly, in February, April, October, and December. Correspondence may be directed to specific editors or to Mu Alpha Theta National Office, 601 Elm Ave., Rm. 423, Norman, OK 73019.

Aim for Higher Goals Andree Winner Urges

"I want to tear down the mental wall that students may have built against mathematics and show them how useful a sound knowledge of mathematics can be."

Tammy Kirkland, Mu Alpha Theta president at East Central Community College, Decatur, MS, penned those words as a candidate for the Richard V. Andree "future teacher" award.

As Andree Award winner getting ready to travel to Mu Alpha Theta national convention in Tampa, Tammy shared these thoughts which she asked be shared with Mathematical Log readers:

"I would like to challenge all Mu Alpha Theta members to continue your interest in mathematics and to continue to excel in your course-work.

"Never be satisfied, always aim for higher goals--but all the while, remain proud of yourself and your accomplishments."

Tammy graduated from Neshoba Central High School with an outstanding student record and special awards in mathematics, science, and band, but with no experience with Mu Alpha Theta, the high school having had no chapter. She was welcomed into the community college chapter on the basis of her first-term grades, and was elected chapter president in her second year. Tammy also was Student Education Association president, Phi Theta Kappa vice-president, scholar's bowling team captain, and President's Council member. She won the Freshman Mathematics Award, 1989 ECCC Outstanding Student Award, and was named to the National Dean's List and All-American Scholar and Presidential Scholar. She plans to continue her studies at Mississippi State University, majoring in Mathematics and minoring in Education, in preparation for her career as mathematics teacher.

In her Andree Award essay presenting her rationale for chosing mathematics teaching as a future career, Tammy identified it as a lifetime ambition--to teach--coupled with a desire to share her favorite subject.

Tammy wrote:

"As a young child, like most little girls, I loved to pretend that I was a teacher. I would seat my dolls all around me and teach them all that I had learned, meticulously imitating my own teachers. I often dreamed of the day when I would have my own classroom with real students. However, unlike most young girls, my dream has not changed.

"I have always wanted to become a teacher. I have often been told that I should become a doctor, a lawyer, or even a scientist. Some have remarked that it seems a waste of my potential to become a teacher; yet, I can think of no better way to use my abilities than by helping others expand their minds.

"Throughout school, mathematics has always been my favorite subject. In fact, I recall a time when I was only in grammar school that after our mathematics teacher explained a new topic and assigned work for us to do, several of my friends would call me over to re-explain the topic. They always told me that I had a special ability to explain topics in a way that they could better understand. This became a trend which continued throughout high school. Mathematics comes so naturally to me that I feel my capacity for it must have been inborn.

"I feel that mathematics is the most important subject in the school curriculum. A person needs a strong mathematics background no matter what he chooses to do in life. As our society becomes more and more technological, that need for mathematical knowledge will become more imperative.

"Yet so many people perform poorly in mathematics. Some mistakenly that mathematics is boring. On the contrary, mathematics can be very exciting. One can find so many ways to be creative in teaching math. I do not want my students to become bored with mathematics. I want to tear down the mental wall that students may have built against mathematics and show them how useful a sound knowledge of mathematics can be. Once students can see a purpose for learning mathematics, I feel that they will develop a great enjoyment of it.

"In summary, I want to become a mathematics teacher because I feel the nation needs more dedicated, innovative teachers for its young people. I want to instill in my students a love for mathematics as well as a love for learning."

Tammy, who tells The Log how much she has enjoyed being a part of Mu Alpha Theta during her community college years, encourages students who are Considering a career in mathematics teaching to submit an Andree Award application. Any who wish to contact Tammy may do so at Route 2, Box 218, Philadelphia, MS 39350. Tammy's Mu Alpha Theta sponsor at East Central Community College has been Mrs. Lois McMullan.

diaLogue

with Log Editor Don Allen

A pleasant if improbable evening with hyenas, eagles, and snakes--mutually devouring critters--has left us with a feeling of some satisfaction, but with more good questions than we started with, as open-ended math problems frequently will.

The hyenas, eagles, and snakes derive, as you might suspect, from the Swaziland math competition, featured in part in February's Log (page 8). Our evening with the African fauna we owe to Sandra Hightower of Skyline High School, Dallas, TX--who wrote requesting--not unreasonaby--the answers to the trio of Swaziland questions.

The first two questions were distinctive but routine, and we'll treat them accordingly. The third, involving animal populations, "opens up" intriguingly, so we'll both provide answers and suggest directions for further investigation.

Here are the three Swaziland problems, with answers and rationales.

1. If it is raining in Mbabane at midnight, will Manzini have sunny weather 72 hours later?

A classic "trick question" in African dress. The sun may never have set on the British Empire (Swaziland, a constitutional monarchy, has long British connections), but any given part of it sure knew night and day! Midnight plus 72 hours equals midnight, so no sun in Manzini!

2. Seven good friends dine in the same restaurant. All are eating there today. However, they do not all eat there every day. The first eats there every day, the second every second day, the third every third day, ..., and the seventh every seventh day. How Many days from today will they all meet in the restaurant?

Let's talk about it.

The seven certainly will be together again in $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$, or 5040 days (something over 13 years), or in any multiple of 5040 days.

But, in fact, they will be together much sooner.

We need not the product but the least common multiple of 1 through 7, since the LCM will be less than the product due to 2, 4, and 6, and 3 and 6 having common factors.

(See "diaLogue," page four)

Group 'Power Question'

FROM PAGE ONE

P4. Find the sides of this P4.

B. Find a P4 with two sides equal and with $d = 25$ that is different from the answer to part IIA. [Note: Two Pn's are not considered different if their sides are equal, but in a different order.]

C. Show that a Pn must exist for all integers $n \geq 3$. [This may be done by describing how to create such a Pn.]

III.

A. For the P3:(a,b,d), $d^2 = a^2 + b^2$. Prove that for the P4:(a,b,c,d), $d^2 > a^2 + b^2 + c^2$.

B. Given the P4:(a,b,c,d). Prove that if $d > 2$, then d must be composite.

C. If all the diagonals of a Pn are integers, we will call it "Super Pythagorean" and denote it by $\overline{P}n$.

1. Show that the area of any $\overline{P}4$ must be an integer. [Hint: One approach might be to first show that the area of any $\overline{P}4$ must be rational, and its perimeter must be even.]
2. Assuming that the area of every $\overline{P}3$ and every $\overline{P}4$ is an integer, show that (for all $n > 4$) the area of every $\overline{P}n$ must be an integer. [You may do this part even if part IIIC1 has not been completed.]

dia LOG ue

FROM PAGE THREE

The required LCM is 3 x 4 x 5 x 7, or 420. The friends will reassemble in 420 days and in every multiple of 420 days.

Now the hyenas, eagles, and snakes:

3. In a field are some hyenas, eagles, and snakes. Every morning every hyena eats one eagle. At mid-day every snake eats one hyena. In the evening every eagle eats one snake. At the end of the third day there was one eagle left and no others. How many of each were there at the beginning of the first day?

Read carefully, draw up a table, watch for number errors, and you'll not go wrong: 13 hyenas, 19 eagles, and 9 snakes. Work forward as a check.

* * *

To see how the Swaziland problem has potential for "opening up," extend your tabulation backward in time for, say, seven days instead of three days, and give thought to the sequences involved. (The "field" gets a bit crowded, but as with counting ancestors back to the n^{th} generation, that doesn't spoil the math of the theoretical problem!) Here's a tabulation through 21 meals, to an initial population of 872 snakes, 1278 hyenas, and 1873 eagles--4023 animals in all. Note how the entries in columns for specific species, for example the "9, 13, 19" of the original problem, turn up [underscored] in the totals column as well. That "1, 2, 3, 4, 6, 9, ..." "total" sequence seems to say it all, but exactly how may be less than clear.

The sequence, though with three "components," serves to recall the classic Fibonacci sequence (1, 1, 2, 3, 5, ...) and its "rabbits" problem, from which so much good mathematics has arisen.

DAY OF 7	DAY OF 3	FEEDING	NUMBER OF ANIMALS SURVIVING FEEDING snakes (s)	hyenas (h)	eagles (e)	total
(0)			872	1278	1873	4023
1		i	872	1278	595	2745
		ii	872	406	595	1873
		iii	277	406	595	1278
2		i	277	406	189	872
		ii	277	129	189	595
		iii	88	129	189	406
3		i	88	129	60	277
		ii	88	41	60	189
		iii	28	41	60	129
4		i	28	41	19	88
		ii	28	13	19	60
	(0)	iii	9	13	19	41
5	1	i	9	13	6	28
		ii	9	4	6	19
		iii	3	4	6	13
6	2	i	3	4	2	9
		ii	3	1	2	6
		iii	1	1	2	4
7	3	i	1	1	1	3
		ii	1	0	1	2
		iii	0	0	1	1

* * *

A different route to exploring the potential of the Swaziland problem would be to retain the "three days, nine meals" feature while allowing the numbers of surviving creatures to vary. The "general case" might involve s surviving snakes, h hyenas, and e eagles.

Following through with this idea, we "open up" as follows:

Let the numbers of snakes, hyenas, and eagles at the end of a given day be, respectively, s, h, and e.

Let the numbers of snakes, hyenas, and eagles at the start of that day, or at the end of the preceding day, be, respectively, S, H, and E.

Then:
- --prior to the evening feeding, there were s + e snakes, h hyenas, and e eagles;
- --prior to the mid-day feeding, there were s + e snakes, h + s + e hyenas, and e eagles;
- --prior to the morning feeding, there were s + e snakes, h + s + e hyenas, and e + (h + s + e), or 2e + h + s eagles.

So $S = s + e$, $H = h + s + e$, and $E = 2e + h + s$.

Let the number of snakes, hyenas, and eagles at the start of the preceding day be, respectively, S_2, H_2, and E_2.

Then, extending the above reasoning,
- --$S_2 = S + E = (s + e) + (2e + h + s) = 2s + h + 3e$;
- --$H_2 = H + S + E = (h + s + e) + (s + e) + (2e + h + s) = 2h + 3s + 4e$;
- --$E_2 = 2E + H + S = 2(2e + h + s) + (h + s + e) + (s + e) = 6e + 3h + 4s$.

Finally, let the number of snakes, hyenas, and eagles at the start of the day before that be, respectively, S_3, H_3, and E_3.

Further extending the above reasoning,
- --$S_3 = S_2 + E_2 = (2s + h + 3e) + (6e + 3h + 4s) = 6s + 4h + 9e$;
- --$H_3 = H_2 + S_2 + E_2 = (2h + 3s + 4e) + (2s + h + 3e) + (6e + 3h + 4s) = 6h + 9s + 13e$;
- --$E_3 = 2E_2 + H_2 + S_2 = 2(6e + 3h + 4s) + (2h + 3s + 4e) + (2s + h + 3e) = 19e + 9h + 13s$.

That is, in effect: if at the end of the third day (in the Swaziland problem) there had been s snakes, h hyenas, and e eagles, then at the beginning of the first day there would have been 6s + 4h + 9e snakes, 6h + 9s + 13e hyenas, and 19e + 9h + 13s eagles. (continued, page six)

LET'S HEAR FROM YOU!

Photos, cartoons, news items, personal notes, problems you pose, ideas you "write up," all make it distinctively your Mathematical Log. Resolve to keep your interests--and your chapter--in the news.

Math of Investment Yields Social Security Insights

by
Ali R. Amir-Moéz
Mathematics Editor

Everyone may enjoy computing the amount paid for social security and may also find out how much it amounts to with a modest interest. We shall study the simple mathematics involved which amounts to the study of geometric progression and a few other ideas.

This note is a review of compound interest which may interest a few young people to study and play with it. When something of interest is in the background, one would put more effort in it.

1. The Compound Interest: Let the Social Security contribution for each year be a. Let the rate of interest be r and compounded k times per year. Then we observe that for each period the rate will be $\frac{r}{k}$. Now the end of the first period we shall have

$$a + a\frac{r}{k} = a\left(1+\frac{r}{k}\right) = a_1. \quad (1)$$

At the end of the second period we shall have

$$a_2 = a_1\left(1+\frac{r}{k}\right) = a\left(1+\frac{r}{k}\right)^2. \quad (2)$$

So after n years we will have $\frac{kn}{}$ periods and the amount for the n years will be

$$A_n = a\left(1+\frac{r}{k}\right)^{kn}. \quad (3)$$

So after n years we have the sum

$$S = A_1 + A_2 + \ldots + A_n. \quad (4)$$

One observes that S is the sum of n terms of the geometric progression

$$\begin{aligned} S &= a\left(1+\frac{r}{k}\right)^k + a\left(1+\frac{r}{k}\right)^{2k} + \ldots + a\left(1+\frac{r}{k}\right)^{nk} \quad (5)\\ &= a\left[\left(1+\frac{r}{k}\right) + \ldots + \left(1+\frac{r}{k}\right)^{nk}\right]. \end{aligned}$$

For simplicity let $\left(1+\frac{r}{k}\right)^k = p$. Then (5) will be

$$S = a[p + \ldots + p^n], \quad (6)$$

and multiplying both sides by p we get

$$pS = a[p^2 + \ldots + p^{n+1}]. \quad (7)$$

Subtracting (6) from (7), we obtain

$$PS - S = (p-1)S = a(p^{n+1} - p) = ap(p^n - 1). \quad (8)$$

Consequently, we get

$$S = a\frac{p(p^n-1)}{p-1}. \quad (9)$$

So for $p = \left(1+\frac{r}{k}\right)^k$ we obtain

$$S = a\frac{\left(1+\frac{r}{k}\right)^k\left[\left(1+\frac{r}{k}\right)^{nk} - 1\right]}{\frac{r}{k}} \quad (10)$$

or

$$S = \frac{ak}{r}\left(1+\frac{r}{k}\right)^k\left[\left(1+\frac{r}{k}\right)^{nk} - 1\right]. \quad (11)$$

Now let us give a modest example. Suppose a person has an income of \$10,000 per year. Usually up to \$37,800 one pays 6.7% of his or her income for social security tax. (Note that this is somewhat out of date.)

So the person of our example pays \$670 per year, that is, if he or she remains with the same income. One is aware that a high capital gets large interest, but we suppose that the person in our example receives the interest of 10% once a year. If he or she starts working at age of sixteen and pays 35 years of social security tax and it is only compounded once a year, that person by (11) will have

$$\begin{aligned} S &= \frac{670}{.10}\left\{1.1[1.1]^{35} - 1\right\} \\ &= (67)(1.1)^{36} - 1.1 = 199,744.96. \end{aligned}$$

Imagine what a person with average income will have to his or her credit.

2. The Continuous Compounding: Let us consider (3), that is

$$A_n = a\left(1+\frac{r}{k}\right)^{kn}. \quad (12)$$

This can be written as

$$A_n = a\left[\left(1+\frac{r}{k}\right)^{\frac{k}{r}}\right]^{nr}. \quad (13)$$

When the continuous compounding is considered, k becomes very large. In this case we say k approaches infinity. In symbol we write $k \to \infty$. (Even though this may sound very impressive, it does not add much to the interest.) It is well known that

$$\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{\frac{k}{r}} = e. \quad (14)$$

Usually e is approximated by 2.7, but one may consider more decimals such as

$$e \cong 2.718281828459045.$$

From (14) we get

$$A_n = ae^{nr} \quad (15)$$

Then for (5) we get

$$\begin{aligned} S &= A_1 + \ldots + A_n = a(e^r + e^{2r} + \ldots + e^{nr}) \quad (16)\\ &= a\frac{e^{(n+1)r} - e^r}{e^r - 1}. \end{aligned}$$

Let us now compute the value of our example using (16). We get

$$S = 670\frac{e^{36(.1)} - e^{.1}}{e^{.1} - 1} \cong 226,111.49.$$

Indeed, it is quite easy to call a bank and ask the question about how much would be the amount when so much a year is put in a savings account with a certain rate. But it is more interesting to compute one's own amount. Imagine if one puts the 7% of an income of \$37,800 per year in the social security and consider a 12% interest compounded continuously, how much does one have after thirty years of work. Why don't you try (16) and use a good hand calculator?

Now let us ask, *"Where does the money go?"* The Social Security payment is quite poor including other benefits such as medicare, etc. Enrolling in medicare costs also another \$14.60 a month which is \$175.2 a year. (This amount is also somewhat out of date.)

3. Questions: Indians received *twenty four* dollars for Manhattan Island in the year 1576. If they had invested that money, by the year 1920, it would have become 8,271,157,608. What rate would bring that much money if it were compounded *continuously?*

What would be the rate if it were compounded once a year?

What would be the rate if it were compounded four times a year?