20 'HANO' CRYPTARITHMS, page 3

THE MATHEMATICAL LOG

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MU ALPHA THETA

CONTEST PROBLEMS OFFER LIVELY CHAPTER AGENDA

Mathematics competitions can be interesting and instructive individual and group experiences for those actually taking part, but have an equally important, if less recognized, educational function in providing a spinoff of good questions for subsequent browsing and discussion. Books are available of contest questions, answers, and outline solutions. Most competitions also provide access to past papers. From time to time, The Mathematical Log also shares, as potential program material, questions from significant competitions to which chapters might not otherwise have access. This Log we tap two especially interesting sources, available to us through exchange arrangements. The first is a twenty-question Alberta High School Mathematics Prize Examination, which we share, essentially as run, below. The problems, all multiple-choice, we leave for chapter discussion, with correspondence most welcome. The second offering, elsewhere in this Log issue, is from a small African kingdom, Swaziland. Select problems that seem best for chapter investigation, and look to a lively meeting featuring alternate methods of solution.

The 20 questions of the 1986 Alberta competition are presented, in essence, below:

1. A square lawn 10 metres by 10 metres contains 10 blades of grass per square centimetre. The total number of blades of grass on the lawn is
(A) 1000 (B) 10 000 (C) 100 000 (D) 1 000 000
(E) 10 000 000 .
2. The symbol * in the inequality $n < n * (n+1)$ is to be replaced by +, −, ., or /. The number of replacements so that the inequality holds for at least one integer n is
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 .
3. If $x^2 + Ax + B = (x - A)(x - B)$ for all numbers x and $A \neq B$, then B is equal to
(A) −2 (B) −1 (C) 0 (D) 1 (E) 2 .
4. An n-sided polygon has sides of lengths 1, 2, 4, ..., 2^{n-1}. The smallest possible value of n (at least 3) is
(A) 3 (B) 4 (C) 5 (D) greater than 5 (E) non-existent.
5. The quadratic polynomials $x^2 + Ax + B$ and $x^2 + Bx + A$, where $A \neq B$, have a common root. The value of A + B is
(A) 1 (B) 2 (C) −1 (D) −2 (E) not determinable .
6. Let T(P) be the temperature at a point P on the circumference of a circle C of radius 100 kilometres centered at Norman, OK, at some given time. T(P) varies continuously with P. If A and B are the end points of a diameter of C, the statement among the following which is always true for at least one choice of A and B is
(A) $T(A) > T(B)$ (B) $T(A) < T(B)$ (C) $T(A) = -T(B)$
(D) $T(A) = T(B)$ (E) none of these .
7. The number of digits in the base ten representation of the number $2^{1986} - 1$ is closest to
(A) 500 (B) 550 (C) 600 (D) 650 (E) 700 .
8. The number of planes of symmetry of a rectangular box of dimensions 1 by 1 by 12 is
(A) 2 (B) 3 (C) 5 (D) 9 (E) none of these .
9. If three dice are rolled, the probability of getting a sum of six is
(A) 3/216 (B) 7/216 (C) 10/216 (D) 13/216 (E) none of these .
10. A 3 by 3 by 3 cube consists of 27 unit cubes. Two subcubes of this cube are different if at least one of them contains a unit cube not contained in the other. The number of different subcubes (excluding the 3 by 3 by 3 cube) is
(A) 8 (B) 29 (C) 31 (D) 35 (E) none of these .

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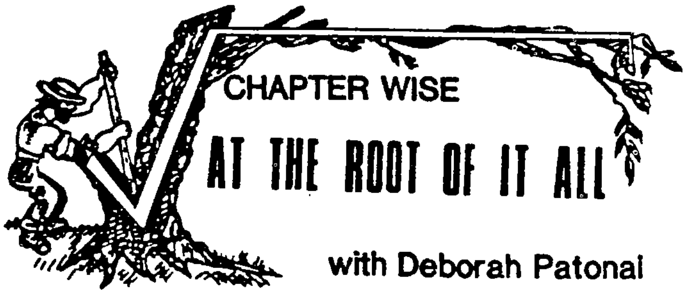
11. If $x^y = z$ and $y^z = x$, then z^x is always equal to
(A) y (B) x^{xyz} (C) y^{xyz} (D) x to the power y^z
(E) y to the power x^z .
12. The closest approximation to $(1.0009)^{1/3} - (1.00009)^{1/3}$ is
(A) 0.3 (B) 0.03 (C) 0.003 (D) 0.0003 (E) 0.00003
13. If x and y are real numbers, the minimum value of $x^2 + 2xy + 3y^2 + 4y + 3$ is
(A) −1 (B) 0 (C) 1 (D) 2 (E) none of these .
14. A sum of three non-zero numbers is eight times the first number, three times the second number, and k times the third number. The value of k is
(A) 13/24 (B) 24/35 (C) 24/19 (D) 24/13 (E) not determinable .

(See "Contest Problems," page 8)



(Activities Editor Deborah S. Patonai calls upon chapters for news and views to share in this regular department. At the Editor's suggestion she provides this issue an overview of her own highly successful Ohio chapter, underlining that these are the kind of "grass roots" Mu Alpha Theta insights that she welcomes from Log readers.)

Deborah Patonai writes:

Last Spring students at St. Vincent-St. Mary High School in Akron, OH, encountered mathematics from all angles. They were exposed to and confronted with mathematics in the classrooms, in the halls, in the student center, and over the PA system. The School's Mu Alpha Theta chapter designated the third week in April as Math Week. Math Week was organized to increase student interest in mathematics as well as to promote mathematics in a useful and entertaining way.

Planning for Math Week began almost a year ahead of time. The main goal of the Mu Alpha Theta Chapter was to generate a variety of math-related activities that would involve the entire student body. Chapter members felt that far too many present-day students are "turned off" by mathematics. Hoping that the opposite might happen, these Mu Alpha Theta enthusiasts set to "turn on" such students. Consequently, an entire week of math activities and contests was carefully planned to involve students at all stages and levels of school mathematics.

So, with a formal proclamation by chapter president Cara Evans, St. Vincent-St. Mary's Math Week began. Cara declared the Week "to be observed in the classrooms of the school community in order to recognize the increased importance of mathematics in our own lives." Throughout the Week, the mathematics theme was everywhere. Math posters with corny math puns adorned halls: "What is an occupied restroom on an airplane? Hypotenuse!" Bulletin boards encouraged "tangents" to "get radical." Morning announcements were preceded by such math songs as "Hip to be Square" and "One." During the Week a math slogan contest was sponsored. The winning slogan by Jennifer Sparhawk of Mu Alpha Theta: "Math isn't just for squares; it's for radicals, too!"

Math Week, at that point, was in full force. On its opening day, all math students viewed the film, "Donald Duck in Mathemagic Land." On the following day students could watch the video, "Square One," in the student center. This day also saw the preliminary round of the "Quickest Calculator in the West" contest. Open to any student who had access to a calculator, this round consisted of a written test that would identify four finalists--one from each grade.

Wednesday of Math Week was another full day of mathematics. All advanced students were invited to hear Dr. Neal Raber, Akron University, on "Amusements in Mathematics." The assembly opened with the "Calculator Shoot-Out Finals." On stage, the four finalists, decked out in cowboy hats and bandanas, solved math problems to the cheers of the crowd. The winner of this event won a calculator!

Thursday was designated Career Day. People from the community, including students' parents, were invited to participate in a Career Fair--describing how mathematics served

...see page 7.

OUTSTANDING SPONSOR RECOGNIZED. Mu Alpha Theta convention in Knoxville was highlighted for many by the presentation of the Harold Huneke Award to Sister M. Scholastica, I.W.B.S., Blessed Sacrament Academy, San Antonio (left). Show with Log Activities Editor Deborah S. Patonai, Sister Scholastica has been a dedicated friend of her school, state, and national organizations, and a familiar figure at conventions. (Don Allen photo)

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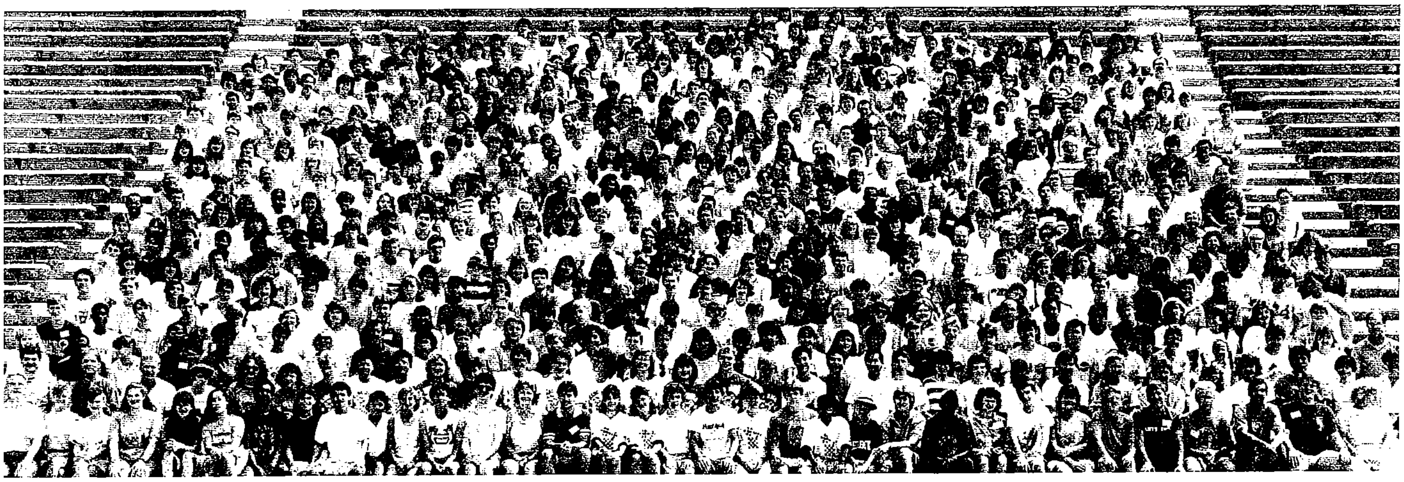
MU ALPHA THETA

The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Mu Alpha Theta, founded in 1957 by Richard and Josephine Andree, is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). The Mathematical Log is published quarterly, in February, April, October, and December. Correspondence may be directed to specific editors or to Mu Alpha Theta National Office, 601 Elm Ave., Rm. 423, Norman, OK 73019.

19th NATIONAL CONVENTION, TAMPA, FL, AUGUST 3-8, 1989

Sponsoring chapters: King High School, Tampa, FL; Berkeley Prep School, Tampa, FL; Plant City High School, Plant City, FL. Registration particulars: Dave Steele, Plant City High School, One Raider Pl., Plant City, FL 33566 (school: 813 754-1541; residence: 754-8306).



CHEERING SECTION for the Volunteers, University of Tennessee football squad? No, Mu Alpha Theta, assembled at the Knoxville stadium at its 18th National Convention for a group photo of "mathematicians in the making" that has become a Convention highlight. It's none none too soon to plan for this year's major event, scheduled for Tampa, FL, August 3-8. Latest Convention information, rates are available from organizers: mailing addresses, telephone numbers are given on your Log masthead, page 2. University of Tennessee photo.

Math Club Fare

CRYPTARITHM CHALLENGES

Scandinavian Master Puzzlist Shares Twenty Math Favorites

By Don Allen

Recreational cryptography and recreational mathematics (two of the more challenging and more rewarding of present-day leisure mental pursuits) intersect, so to speak, in the exotic realm of the cryptarithm or alphametic, a remarkable puzzle form that can be fun either to solve or to create. Letters substitute for the digits of mathematical examples. The result can be a meaningful cluster or words concealing a unique set of numerical equivalences. Most of us have seen and perhaps tried the cryptarithm purporting to be a college student's message home:

```
  S E N D
+ M O R E
---------
M O N E Y,
```

an addition cryptarithm, base ten, a familiar though not particularly fine illustration of the puzzle form. More worthy of attack and of subsequent logical analysis is the 1940's classic of U.S. high school teacher Alan Wayne,

```
  F O R T Y
      T E N
+     T E N
-----------
  S I X T Y,
```

which involves all ten decimal digits, has a unique solution, and lends itself to a purely logical, rather than trial-and-error approach. (For Y + N + N = Y in the ones column, N has to be Ø ... or else 5. For T + E + E = T in the tens column, E by assumption not equal to N, there can be no carrying from the ones column, so ....) In his 2nd Scientific American Book of Mathematical Puzzles and Diversions (Simon and Schuster, 1961), mathematics popularizer Martin Gardner shares a full logical consideration of this remarkable 'rithm (p. 243). Quite good cryptarithms can be found on a regular basis if you know where to look for them in the periodical literature. "Problems" departments of School Science and Mathematics and The Journal of Recreational Mathematics offer occasional real challenges. (In this connection, N.J. Kuenzi and Bob Prielipp, Cryptarithms and Other Arithmetical Pastimes [School Science and Mathematics Association, 1979] is a useful SSM compilation.) Some of the hardest and many of the best new cryptarithms appear in The Cryptogram, published six times a year by the venerable American Cryptogram Association (P.O. Box 6454, Silver Spring, MD 20906). Also available from ACA are good guides as to "how to solve." Among the more

(See "Cryptarithm Challenges," page 4)

Cryptarithm Challenges

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imaginative of ACA cryptarithms in recent years, we've felt, have been those contributed by HANO--pseudonyms are usual within the cryptographic fraternity. HANO is, the ACA membership roster informed us, a Swede named Henning Orlando. Correspondence with Stockholm put us in touch with a retired architect of diverse and remarkable talents--who lists mathematics among his interests and hobbies in retirement, and who has proved a good friend to Mu Alpha Theta.

Mathematical Log readers have, over the years, responded well to cryptarithm and related challenges. (The KNOXVILLE TENNESSEE convention cryptogram--October Log--was promptly solved by the Mu Alpha Theta chapter at North Garland High School, Garland, TX, Kevin Kong, the chapter president, has written to report.) Accordingly, we were delighted when, in response to a Mathematical Log request, Henning Orlando granted Mu Alpha Theta the right to select and share our choice from his scores of acclaimed cryptogram "constructions." He even allowed us to preview and use some highly original, unpublished efforts. We feature twenty HANO constructions this issue. We also invited a photo and biographical details. HANO has responded with a doodle (drawing is another of his lifelong hobbies), and notes on a remarkable national and international career.

HANO was born in Paris in 1914, he tells us. His father was Swedish, his mother German. He began his schooling in France, then went on to graduate as an architect from the University of Technology in Stockholm. His work extended to town planning, and the designing of housing and of such structures as hotels and schools. "The last 13 years of my career I was director of the Swedish Building Standards Institution, principally working with international standardization in the building field within ISO [the International Standards Organization]," he writes. It was ISO business that brought HANO on his one visit to Canada, when he participated in a Toronto meeting on modular construction. He at that time met a dozen leading figures in recreational cryptography in Toronto and New York. Writes HANO: "You will have to content yourself with a self-portrait. However, it does not show all my wrinkles."

A well-constructed cryptarithm--and HANO's certainly are--has one letter for each digit, and uses it consistently throughout. The letters are used as digits, not as in algebra: with A = 5, B = 7, ABBA would denote the number 5775. All HANO's 'rithms are "keyed"--that is, the letters assigned to digits, when arranged in a stipulated order, read out as one or more words. Thus, in the first HANO construction below, "Two words, Ø-9" indicates that the letters will spell out two words when written above their associated digits in the order zero to nine, a check on (or clue to) the results.

Twenty HANO constructions follow. They could make for a fine chapter meeting, or more than one evening of pleasurable individual exploration. You can reach HANO at Olaus Petrigatan 2, S-115 34 Stockholm, Sweden, if you want to say "thank you" or to discuss your results.

HANO-Ø1. Additions. Two words, Ø-9.

AL + IT = TG. EF + NL = CS. IG + LE = EC.
FA + NG = TS.

HANO-Ø2. Repeated multiplications. Two words, Ø-9.

C x N x L = UIH. O x A x G = UMO.

HANO-Ø3. Repeated multiplications. Two words, Ø-9.

H x O x E = YEB. T x N x R = YUT. A x H x R = UER.

HANO-Ø4. Distributed multiplications. Two words, Ø-9.

E x (M + B) = T x (L + E).
S x (I + U) = H x (R + I).

HANO-Ø5. Long division. One word, Ø-9.

RESTLESS divided by TROT = LOAN; - ANELO = SHASE; - ENCNS = AHSNS; - HLTTL = SNAAS; - SSLOR = ATLR.

HANO-Ø6. Exact long division. One word, Ø-9.

UUUUUUUU divided by ECM = UMEMT; - ECM = MCEU; - LICS = ATLU; - AILI = LSTU; - LICS = NALU; - NALU = zero.

HANO-Ø7. Repeated roots. Two words, Ø-9.

$\sqrt{EAGRFMRDA}$ = DEGUA. $\sqrt{DEGUA}$ = AUS. $\sqrt{AUS}$ = GG - ID.

HANO-Ø8. Formal square root. One word, Ø-9.

YR PI AE gives root IGC; - GS = IPI; - YIR = AEAAE; - AASPG = ANPS.

HANO-Ø9. Equations. One word, Ø-9.

D x DS = EHA. O x TR = DGA. N x RE = SNN.

HANO-1Ø. Equations. Three words, Ø-9.

W^{D+I} = TFLE. D^{H+N} = NIWO.

HANO-11. Powers. Three words, Ø-9.

D^{T} = TFT. M^{F} = FJEN. I^{T} = USI.

HANO-12. Powers. One word, 9-Ø.

T^{P} = ALMT. O^{T} = AMSO. I^{B} = MAUB.

HANO-13. Differences of powers. Three words, Ø-9.

$Y^{B} - B^{Y}$ = AIOO. $O^{T} - T^{O}$ = SIEE. $U^{T} - T^{U}$ = BIOD.

HANO-14. Equations involving powers. Two words, Ø-9.

$S^{W} - I^{F} = W^{I} + U^{L}$. $W^{R} = G^{F} + F^{O}$. $W^{S} = E^{F}$.

HANO-15. Equations involving powers. Four words, 1-Ø.

$A^{\frac{BT}{C}}$ = UBA. $U^{\frac{IC}{O}}$ = ETE. $T^{\frac{BN}{Y}}$ = BYO.

HANO-16. Differences of powers. One word, Ø-9.

$A^{M} - U^{N}$ = RLE. $I^{R} - R^{A}$ = OTT.

HANO-17. 3 x 3 magic square. Two words, Ø-9.

CL	RO	OR
EK	HS	AA
KE	KB	SH

All rows and columns and both diagonals add to LRK.

HANO-18. 5 x 5 magic square. Two words, Ø-1.

TI	KT	NS	OB	UL
CN	IO	CC	UK	TB
IU	OL	UI	SC	CS
OK	UB	TL	KO	IT
NC	TK	KU	NN	OI

All rows and columns and both diagonals add to TKU.

HANO-19. Trigonometric approximations. Two words, Ø-9.

tan RX^{o} = C.UXR tan LU^{o} = X.CLE tan NS^{o} = R.GGA
RX + NS = LU

HANO-2Ø. Trigonometric equivalences. One word, Ø-9.

cos NE^{o} + cos DG^{o} = cos AE^{o}

sin SH^{o} + sin AH^{o} = sin NH^{o}

cos RR^{o} + cos TS^{o} = cos R^{o}

sin ST^{o} + sin ET^{o} = sin OT^{o}