THE MATHEMATICAL LOG

Volume 33, Number 4

December 1989

MU ALPHA THETA

High School Contest

Latest Santa Clara Questions Perpetuate Fine Tradition

(Maintaining the Mathematical Log tradition of sharing with chapters the questions of outstanding contests and competitions, we offer in its entirety the Santa Clara University 31st Annual High School Mathematics Contest. Mu Alpha Theta thanks Professor Gerald L. Alexanderson for making these questions available at the Log's request. --Ed.)

1. A point P is located in the interior of a rectangle so that the distance from P to one corner is 5 ft, to the opposite corner 10 ft, and to a third corner 14 ft. How far is P from the fourth corner?

2. If $(1 + x + x^2 + x^3)^n$ is multiplied out and simplified, what is the coefficient of
   (a) x, for $n = 2$?
   (b) x^3, for $n = 2$?
   (c) x, for $n = 4$?
   (d) x^6, for $n = 4$?

3. Find the remainder when $x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by $x^2 - 1$.

4. Find all solutions of the equation

$$4\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) - 7 = 0.$$

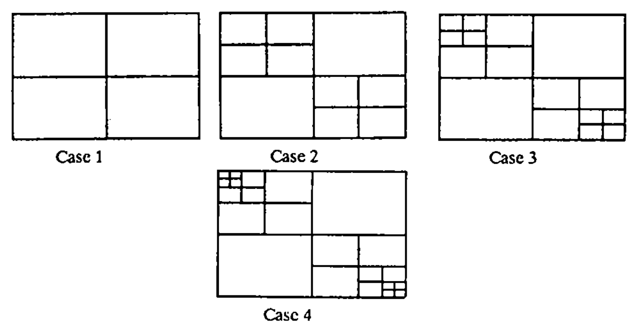
5. For which integers x is it possible to find an integer y such that

$$x(x + 1)(x + 2)(x + 3) + 1 = y^2 ?$$

Prove your conjecture.

6. A man is $\frac{3}{8}$ of the way across a narrow railroad bridge when he hears a train approaching behind him at 60 miles per hour. If he runs in either direction (at the same speed) he barely escapes being hit by the train. What is the slowest speed at which he could run and live to tell about it?

READ ABOUT 1989 ANDREE AND HUNEKE AWARD WINNERS IN DEBORAH PATONAI PHILLIPS' COLUMN--IN YOUR CURRENT TALL TIMBERS SUPPLEMENT.



7. In Case 1, there are 9 rectangles of all sizes (some overlapping). How many rectangles of all sizes are there in
   (a) Case 2?
   (b) Case 4?
   (c) Case 6?
   (d) Case n?

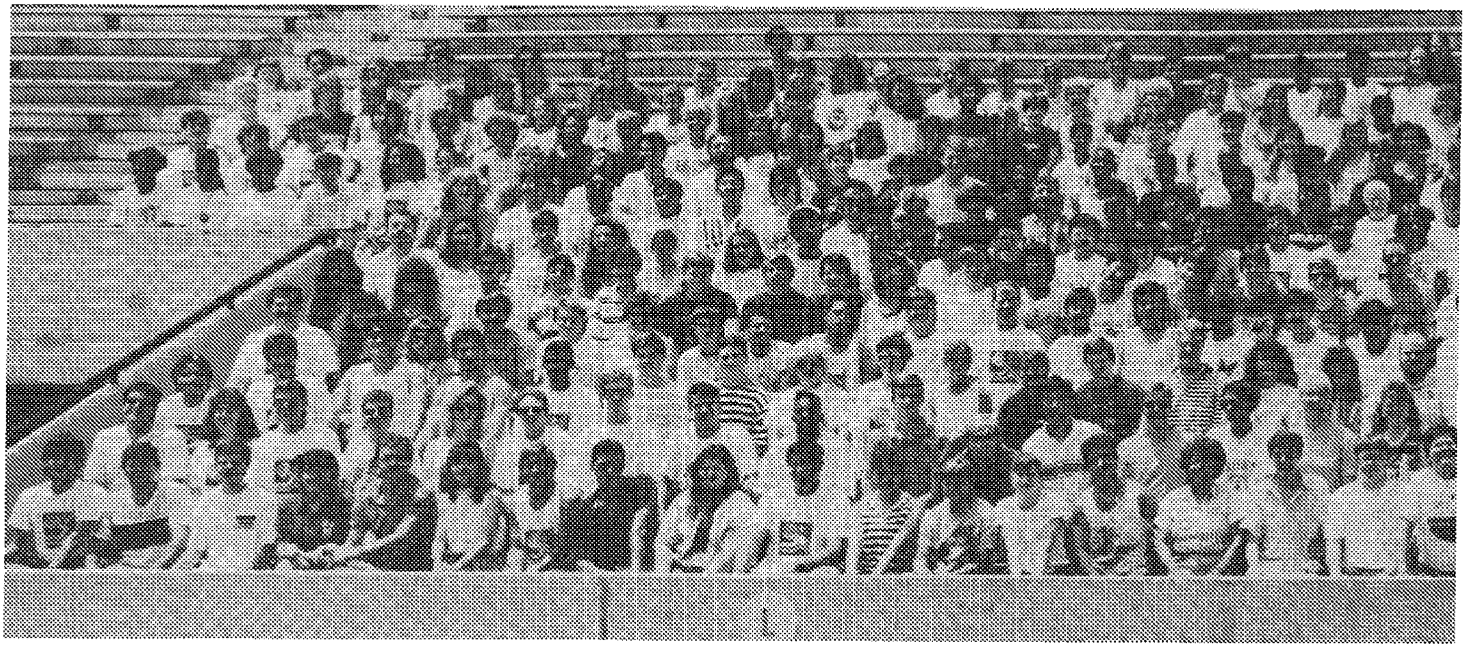
'FRIENDS' GROUP INAUGURATED

Mu Alpha Theta president Dr. Pamela J. Drummond announced at Tampa Convention the decision of the Governing Council to inaugurate an organization, Friends of Mu Alpha Theta, for professionals who would like to support Mu Alpha Theta but whose duties do not fall within the sponsor/chapter structure.

Members of Friends of Mu Alpha Theta would be entitled to all privileges of Mu Alpha Theta affiliation except the right to vote. They would receive The Mathematical Log as individual subscribers, selected current publications, and all new publications.

Annual dues were set at $10.

Further information is available from National Office. News of the formation of Friends of Mu Alpha Theta was to be disseminated through N.C.T.M., N.C.S.M., and M.A.A. publications.



MATHEMATICIANS OF TOMORROW, and leaders in diverse walks of life, are included (we'll wager) in the well over 500 Mu Alpha Theta members photographed at Tampa Stadium on the occasion of Mu Alpha Theta's 19th National Convention in August. Inaugurated at St. Louis convention,

'SYNTHETIC MULTIPLICATION' PROPOSED BY MEMBER

(With the encouragement of sponsor John Wells, student Jeremy Murdock of Sehome High School, Bellingham, WA, approached Log editors at National Convention in Tampa with his idea of "synthetic multiplication." What follows is Jeremy's on-the-spot response to the Log invitation that he share his worthwhile math insights. Ed.)

By Jeremy Murdock

While studying properties of synthetic division, I was struck by this thought: most, possibly all, algorithms can be executed both forwards and backwards--we have, for example, procedures for multiplication and division. Why then do we perform synthetic division, but not synthetic multiplication? Well, now we can! The format I'm suggesting is identical, but the process, different. "Synthetic multiplication" is the name I've given to the technique.

With synthetic multiplication, you can find a polynomial with given roots in a matter of seconds. This is all done with multiplication of coefficients. Personally, I feel that an example can be the best way to communicate an algorithm, so that is how I'll proceed.

Given a list of roots, let's say

$$-\frac{2}{3},\ 2,\ 3,\ -1,$$

we must put them in the form of a factored polynomial equation:

$$(2x + 3)(x - 2)(x - 3)(x + 1) = 0.$$

Next, we place the coefficients of the factor of our choice in the same format as used in synthetic division. My choice of factor is $2x + 3$:

| 2 + 3 |

After placing the coefficients in synthetic form, we multiply the coefficient of A_{n-1} by the negative of the next root, and add the product to A_n to get B_n. Example:

−2	2 +	3	0
	(−2•0)	(−2•2)	(−2•3)
	+2	+3	+0
	=	=	=
	2	−1	−6

$$(-R \times A_{n-1}) + A_n = B_n$$

The complete procedure for the roots $-\frac{2}{3}$, 2, 3, −1, looks like this:

−2	2 +	3	0		
−3	2	−1	−6	0	
1	2	−7	−3	18	0
	2	−5	−10	15	18,

giving $2x^4 - 5x^3 - 10x^2 + 15x + 18 = 0$.

The most obvious advantage to this algorithm is speed, but other advantages might become evident if one were to apply it to another field.

'Timbers' to Offer 1990 Preview

Planned meetings with Ronald Vavrinek, Mu Alpha Theta sponsor at Illinois Mathematics and Science Academy, and other adults and students currently active in the planning of Mu Alpha Theta's 20th National Convention in 1990, should permit exclusive insights into the forthcoming university-based convention, and we're holding space in December's Tall Timbers. Student presentations (papers) will be particularly welcome at this convention, we understand, and now could be the time to start preparation if you'll be attending.