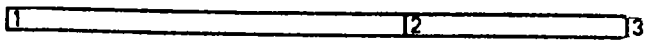
MU ALPHA THETA

Convention / Back to School Issue

THE MATHE-MATICAL LOG

Volume 32, Number 3

October 1988



'Power Questions' Reward Cooperative Efforts

Working as a team to solve a tough problem is a featured approach in national and international mathematics competitions, including some contests at Mu Alpha Theta conventions. Accordingly, good practice problems tend to be more than welcome at chapter level, and Harry D. Ruderman, long-time friend and supporter of Mu Alpha Theta, has been sharing ARML "power questions" and solutions (see December 1987, February 1988 Logs). Featured this issue are to trio of "power question" problems of the 1988 ARML contest.

The three related diagrams are reproduced (see pages 1, 2). Each problem refers to the coordinate axes in its diagram, and in each problem point P is at (0, 4).

Reader comments are welcome. Solutions will be carried in a future Mathematical Log.

I.

At the point (2,0) a vertical segment of length 2 is drawn as shown; call its top point Q_1. $\overrightarrow{PQ_1}$ meets the x-axis at $x = x_1$. A new vertical segment of length 2 is similarly drawn at $x = x_1$; call its top point Q_2. $\overrightarrow{PQ_2}$ meets the x-axis at $x = x_2$. This process is continued as shown.

(a) Compute x_1.

(b) Derive a formula for x_n in terms of x_{n-1}. Show your derivation.

(c) Compute the values of x_2, x_3, and x_4 as integers.

(d) Formulate a conjecture expressing x_n explicitly as a function of n.

II.

A square of side 2 has its base on the x-axis and its lower left vertex at the origin; call its upper right vertex Q_1. $\overrightarrow{PQ_1}$ meets the x-axis at $x = x_1$. A new square of side 2 is drawn with its base on the x-axis and its lower left vertex at $x = x_1$; call its upper right vertex Q_2. $\overrightarrow{PQ_2}$ meets the x-axis at $x = x_2$. This process is continued as shown.

(a) Compute x_1.

(b) Derive a formula for x_n in terms of x_{n-1}. Show your derivation.

(c) Compute the values of x_2, x_3, and x_4 as integers.

(d) Formulate a conjecture expressing x_n explicitly as a function of n.

(See "Power Questions," page 2)

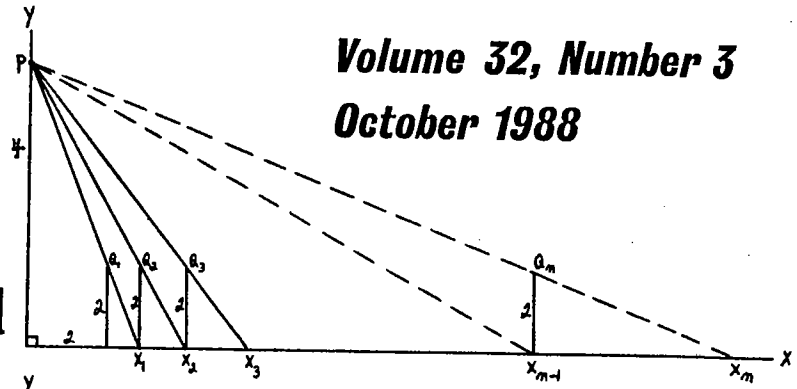


Figure for Power Question I

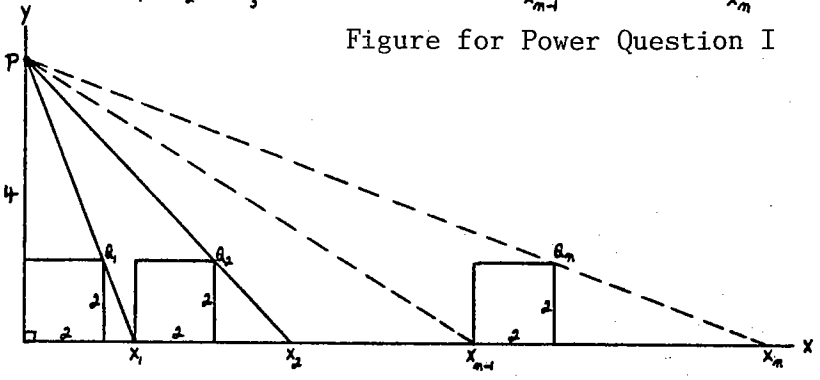


Figure for Power Question II

CONTEST

... CONVENTION CHALLENGE, SEE PAGE THREE

'Power Questions'

... FROM PAGE ONE

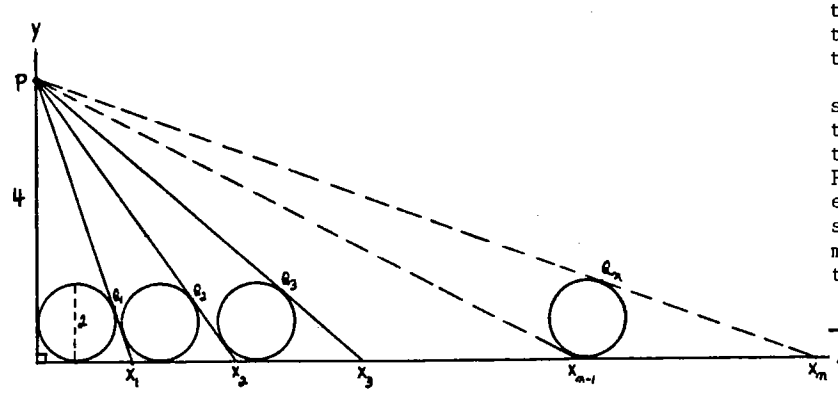


Figure for Power Question III

III.

A circle of diameter 2 is drawn tangent to the x and y axes as shown; another tangent is drawn to the circle from P, meeting the circle at point Q_1 and the x-axis at $x = x_1$. A new circle of diameter 2 is drawn tangent to the x-axis and to $\overline{PQ_1}$ as shown; another tangent is drawn to this circle from P, meeting this circle at point Q_2 and the x-axis at $x = x_2$. This process is continued as shown.

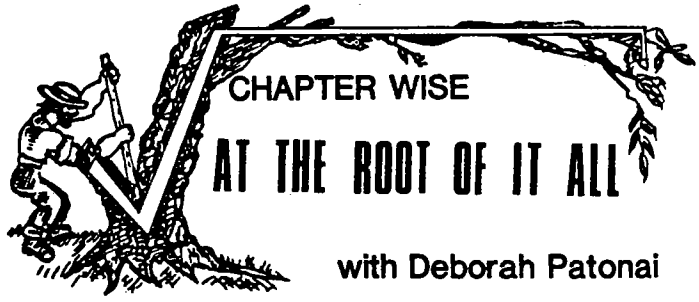
(a) Compute x_1. A formula that may be of use: The area of a triangle (A) can be expressed as $A = rs$, where s is its semi-perimeter, and r is the radius of the inscribed circle.

(b) Derive a formula for x_n in terms of x_{n-1}. Show your derivation.

(c) Compute the values of x_2, x_3, and x_4 in the form $\frac{a}{b}$, where a and b are integers.

(d) Formulate a conjecture expressing x_n explicitly as a function of n.

(e) Prove your conjecture.



Activities Editor Deborah S. Patonai welcomes news, views, and other insights into Mu Alpha Theta chapters and personalities. Look for Debbie at Convention, and drop her a note at her school. You'll be glad you did.

For our back-to-school Log, Debbie writes:

Each August, those Mu Alpha Theta members fortunate enough to attend National Convention are given the opportunity to compete mathematically with some of the best young math minds in the country. They also--as importantly--have the chance to meet other students who share their interests. They attend the planned outings, enjoy the convention, and then go home. At the close of the convention, however, a few lucky Mu Alpha Theta members who attend Berkeley Prep High School in Tampa, FL, embark upon another adventure. Under the direction of their sponsor, Thom Morris, they tour the country!

This Mu Alpha Theta chapter, not unlike many others, works hard throughout the year. Involved with fundraisers, this chapter holds car washes, sells donuts, runs spaghetti dinners, and helps operate the school lunch program. When these students are not occupied with fundraisers, they are busy wrestling with math in order to compete in local and state contests. By participating in such competitions, and through fundraising efforts, Berkeley students earn the right to attend National Convention. In addition to Convention attendance, they also earn the right to take part in the activities which Berkeley Prep arranges for after the Convention.

Although Berkeley Prep had had an active part in several previous National Conventions, the school tradition of organizing a vacation trip for after the convention began with the Honolulu convention. Put under the spell of this tropical paradise, Berkeley Prep students asked if they could stay over and see the Islands. Plans, preparations, and arrangements were thought through, and finalized. After the Convention, as agreed, Berkeley students "recup-

(Continued on Page Five)

THE MATHE-MATICAL LOG

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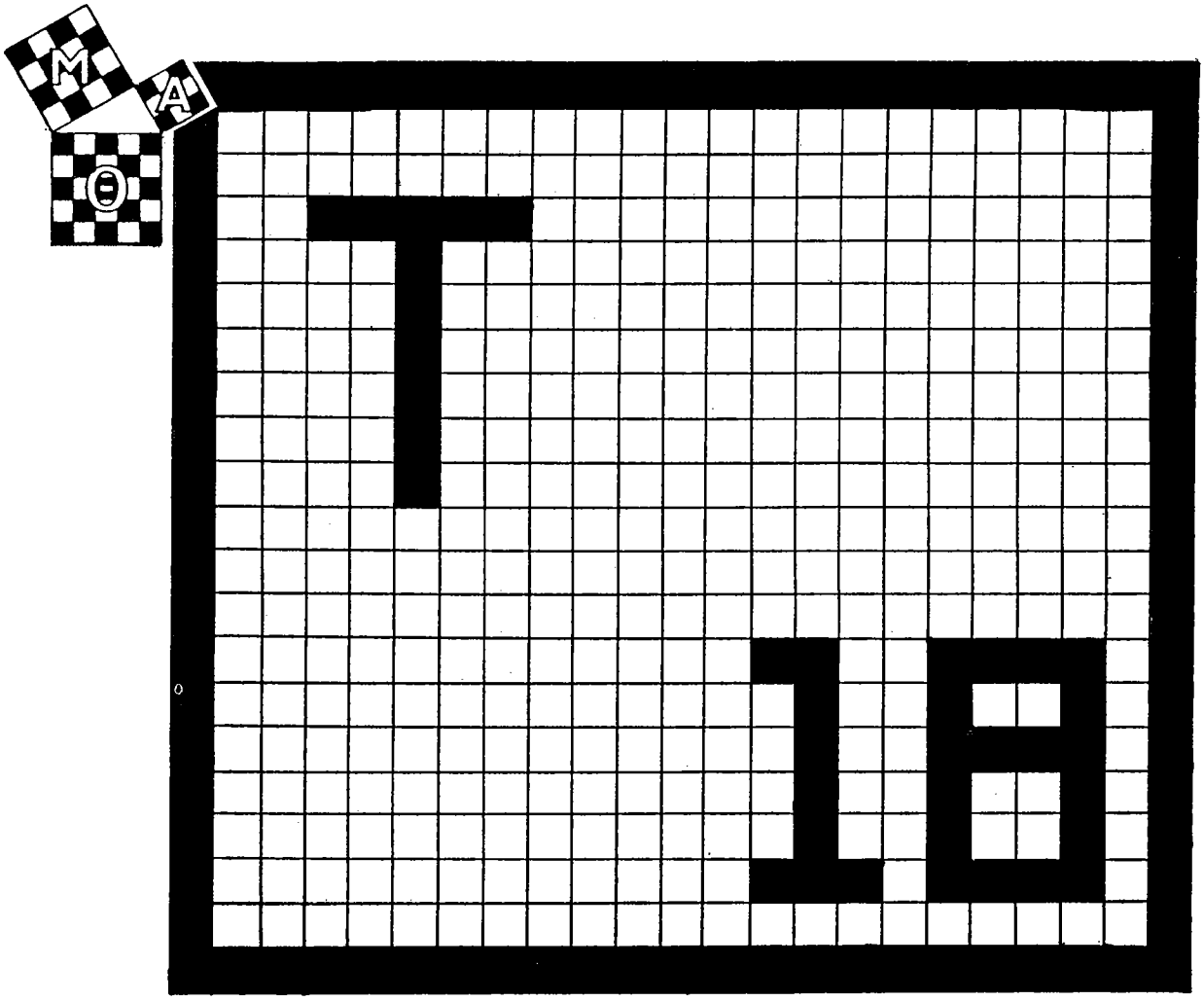
Reach for a Pencil

NEW FALL CONTEST



by Don Allen

Your Log traditionally salutes the Mu Alpha Theta convention and the back-to-school season with a contest challenge to add fun to within-chapter and between-chapter competitive activity. This year's contest, a "math-pack" crossword challenge, marks our 18th National Convention in Knoxville, where it is to make its debut, and builds on a series of highly popular competitions which culminated in the New Orleans contest of 1984. Again this year the Mathematical Log reader is provided with a distinctive grid, and invited to choose math words to maximize his "word packing" score according to a challenging set of letter "values."

The distinctive 19 x 21 grid, with "T" for Tennessee and "18" for our 18th Convention, provides 355 accessible squares. Filled (colored) squares and inaccessible interior parts reduce symmetry and add to the challenge. "Math words," terms used in any branch of mathematics, are to be entered, horizontally (left to right) or vertically (top to bottom), one letter to an unoccupied cell. Words should be of three or more letters, no capitalized words or abbreviations, and--to maximize the fun and the challenge--no root-word may be used more than once. Words may

"cross," as in a crossword puzzle--in fact, all words must be so connected (there should be no "isolated" words or clusters of words). However, in every other sense, individual words must be "separated" (an empty—or colored—cell must "come between" words, horizontally or vertically, though not necessarily diagonally). The letters of words score the points. The object of the activity is to obtain the highest possible score.

1988 LOG "MATH PACK" LETTER VALUES

A - 3	H - 4	N - 4	T - 3
B - 8	I - 3	O - 3	U - 6
C - 6	J - 9	P - 7	V - 8
D - 5	K - 8	Q - 8	W - 7
E - 2	L - 5	R - 5	X - 8
F - 8	M - 6	S - 1	Y - 7
G - 7			Z - 9

Letters of words entered in grid words will score points on a sliding scale (see inset), higher point values being identified with letters deemed less common in English mathematical usage. Thus, E rates 2 points, T rates 3, ..., and J and Z rate the maximum, 9. Plurals may be used, but S rates only 1 point, so there may be better ways of achieving record scores. Note that words are counted, and their letter values added. In effect, any letter common to two words (that is, representing their grid intersection), is counted twice. Accordingly, the word QUOTIENT, 8 letters, would score 8+6+3+3+3+2+4+3, or 32 points.

The highest total point score wins. It's as simple as that. In the event of a tie, the solution achieving the maximum total using the fewest possible words will be declared the winner.

SCORING 849 ON DEMONSTRATION "MATH PACK"

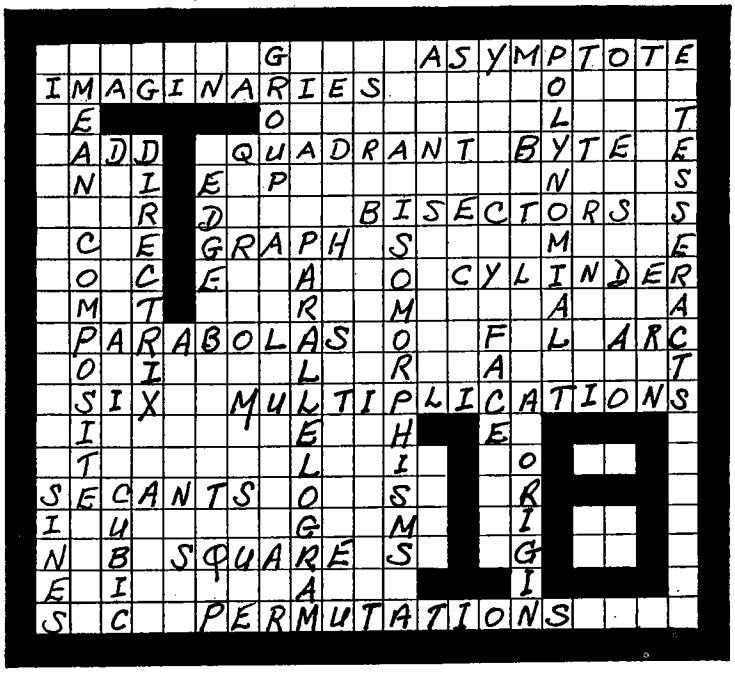
Horizontal words (457 points):	Points
ASYMPTOTE 3+1+7+6+7+3+3+3+2	35
IMAGINARIES 3+6+3+7+3+4+3+5+4+2+1	41
ADD 3+5+5	13
QUADRANT 8+6+3+5+5+3+4+3	37
BYTE 8+7+3+2	2Ø
BISECTORS 8+3+1+2+6+3+3+5+1	32
GRAPH 7+5+3+7+4	26
CYLINDER 6+7+5+3+4+5+2+5	37
PARABOLAS 7+3+5+3+8+3+5+3+1	38
ARC 3+5+6	14
SIX 1+3+8	12
MULTIPLICATIONS 6+6+5+3+3+7+5+3+6+3+3+3+3+4+1	61
SECANTS 1+2+6+3+4+3+1	2Ø
SQUARE 1+8+6+3+5+2	25
PERMUTATIONS 7+2+5+6+6+3+3+3+3+3+4+1	46

Vertical words (392 points):	Points
SINES 1+3+4+2+1	11
MEAN 6+2+3+4	15
COMPOSITE 6+3+6+7+3+1+3+3+2	34
CUBIC 6+6+8+3+6	29
DIRECTRIX 5+3+5+2+6+3+5+3+8	4Ø
EDGE 2+5+7+2	16
GROUP 7+5+3+6+7	28
PARALLELOGRAM 7+3+5+3+5+5+2+5+3+7+5+3+6	59
ISOMORPHISMS 3+1+3+6+3+5+7+4+3+1+6+1	43
FACE 8+3+6+2	19
ORIGIN 3+5+3+7+3+4	25
POLYNOMIAL 7+3+5+7+4+3+6+3+3+5	46
TESSERACTS 3+2+1+1+2+5+3+6+3+1	27

Some suggestions as to strategy: two skills are involved. The first skill is to identify a wide range of suitable "math words," noting those of high total point scores (PARALLELOPIPED = 7+3+5+3+5+5+2+5+3+7+3+7+2+5, or 62) or, possibly as importantly, of high "mean letter value." Thus, CUBIC, adding to 4Ø for a mean value of 4Ø/5 or 8.00, could be worth trying to work in. The second skill is to "weave" the better words into a criss-crossing "connected" pattern, normally starting out with "good" words of greater length. Short words can serve to fill the gaps.

You'll need an umpire for word admissibility, and a good rule is that to use a "math word" you must be able to explain, in simple terms, what it means (TESSERACT?).

Open to groups and individuals within Mu Alpha Theta, our "Tennessee 18" Math-Pack Competition will (like its forerunners) extend through three "phases." Phase I, where the premium likely will be on speed, will be the Knoxville "on-site phase," with a modest prize for the best solution handed to the Editor by the Convention closing session. Phase II, the "cross-country phase," will recognize superior achievement at school and chapter level, through 30 November postmark deadline. Phase III, our "championship phase," will call upon all and sundry to outdo Phase I and II accomplishments, as reported in December and February Logs.



A SCORE OF 849 on this test game is no record setter, but does illustrate correct play. Note the range of terms deriving from various mathematical branches (GRAPH, algebra, statistics, combinatorics; SECANTS, geometry, trigonometry), BYTE (computer science). Note, too, the interweaving of words, and the interconnectedness of all parts. QUADRANT and EDGE are connected, as they must be, but indirectly, through GROUP, IMAGINARIES, MEAN, ADD, DIRECTRIX, PARABOLAS, PARALLELOGRAM, and GRAPH. The inset shows the "scoring" of 28 "math words," 15 horizontal and 13 vertical, to the 849 sum. You can do better. Share your best results.

KNOXVILLE 'RITHMS

KNOXVILLE	KNOXVILLE
+ TENNESSEE	- TENNESSEE
LKVSIKEVN	OKKVISXTV

A further Knoxville challenge, for Convention-goers and for those who solve at home. How elegant a solution can you come up with to this Knoxville cryptarithm pair? Can you show logically the values of the ten letters (each a base-ten digit) that will make this addition and subtraction simultaneously true? Let's hear from you! H.D.A.

From Log Archives

OTHER LA PORTE CITY PHOTOS ... PAGE SIX

Chapter Thrives in 'Service' Role

CHAPTER IN ACTION! The Mathematical Log welcomes photos of Mu Alpha Theta chapters "in action." Photos normally should be timely, sharp, prefereably black-and-white, with individuals identified. Share your pictures--and your stories--with Activities Editor Deborah S. Patonai.

A significant role of the Mu Alpha Theta chapter in many a school community is one, broadly, of service. Such service to student body may take the form of assemblies, scholarship sponsorship, fund raising, tutoring service, or sharing of mathematical interests and insights. From Mathematical Log archives we feature a photographic record of Mu Alpha Theta enthusiasts from our chapter in La Porte City, IA, taking part in a highly successful "Elementary Math Field Day" for Grade VI students. Our records suggest that the students who organized and participated in the event will have gone on to college and to careers--but "once Mu Alpha Theta, always Mu Alpha Theta," as National Secretary-Treasurer Harold Huneke would emphasize--and the Grade VI's may now be Mu Alpha Theta members... the lesson lives on--that Mu Alpha Theta can serve and promote the quality of school life, in diverse ways.

A GOOD PROBLEM! La Porte's Deanna Emberton presides as Grade VI students learn the satisfaction of "opening up" a non-routine problem in mathematics.

SHAPE AND FORM. La Porte chapter's Brenda Walker introduces Grade VI students to the fascination of 3-D skeletal models at the school's Mathematics Field Day.

ORIGAMI, paper folding elevated to an art form, is presented to Grade VI La Porte students by Diane Slick. The Japanese craft is rich in concepts of modern Geometry.

AT THE ROOT OF IT ALL

... FROM PAGE TWO

erated" for three days on Waikiki Beach. Then they boarded a small prop plane that flew them to Hawaii's five major islands. The plane tour ended on Kauai. Once on that island, the group rented a station wagon, piled everyone on board, and headed for Waimea Canyon, Hawaii's Grand Canyon. At the Canyon, the group pitched tents and spent the next five days camping and climbing all over this beautiful Hawaiian paradise.

After Hawaii came the National Convention in Miami. Florida has a lot of attractions to offer the outsider, but the Berkeley Prep students had visited them all. Once again, plans were made for another dream vacation. Resting for a few days in Miami Beach after the Convention, chapter members then boarded a cruise ship bound for Mexico. For the next five days, Berkeley students cruised the Caribbean, visiting Cozumel and Mexico.

Mu Alpha Theta's 1987 Convention in Seattle presented the Florida chapter with so much country that they were not quite sure just what they wanted to do. Their original hope was to fly to Alaska from Seattle, then tour that state. However, their expectation of how much money they could raise during the school year proved a little high. They could not budget for the Alaskan tour, but did come up with an attractive alternate plan. After Convention they visited Vancouver and Vancouver Island--recalling especially the Butchart Gardens. Upon returning to Seattle they travelled to Mount Rainier--enjoying a ten-hour hike up the mountain. This post-Convention tour package totalled five days.

Next on the agenda, of course, has been Knoxville Convention. The Berkeley chapter has had lots of ideas as to what do do after this Convention. Tentative plans, we understand, have been to travel to North Carolina and to take a river raft down the rapids of the Nantahala River.

"All this sounds extravagant, yet it really is not," points out sponsor Thom Morris. With all the fundraising the chapter does throughout the year, this Math Club is able to pay for 85%-90% of a student's total cost for travel, convention fees, and the trip afterwards. Thom explains:

"We work hard all year raising money, doing math at competitions, practicing for and competing at National Convention. These trips make it all worthwhile."