

Concept Extended to Consider Polar Conic of Pentagon

by
Ali R. Amir-Moéz
Mathematics Editor

The concept of pole and polar with respect to circle has already been generalized to the pole and polar with respect to a conic [1]. Also the polar circle of a triangle has been defined in [2]. In this note we study the polar conic of a pentagon, and show its existence and uniqueness.

1 Pole And Polar With Respect To A Conic

Let

$$ax^2 + 2bxy + cy^2 + 2px + 2qy + r = 0 \quad (1)$$

be the equation of a conic section and $P(x_0, y_0)$ be a point (Figure 1). Let

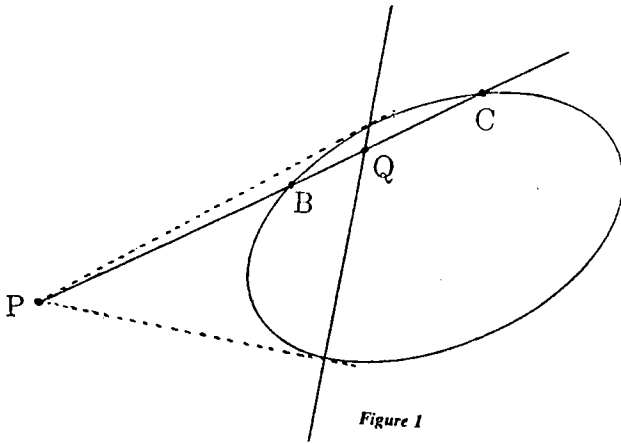


Figure 1

a straight line through P intersect (1) at B and C . Then the locus of Q , the harmonic conjugate of P with respect to B and C , is a straight line called the polar of P with respect to the conic.

Proof: Since Q is the harmonic conjugate of P with respect to B and C , we must have

$$\frac{PB}{PC} = -\frac{QB}{QC} \quad (2)$$

Let

$$\begin{cases} x = x_0 + tl \\ y = y_0 + tm \end{cases}$$

be a set of parametric equations of a line through P . It is clear that $t = 0$ corresponds to P . Let t_1, t_2 , and t correspond to B, C , and Q respectively. Then (2) implies that

$$\frac{2}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

or

$$t = \frac{2t_1t_2}{t_1 + t_2} \quad (3)$$

From the set of equations

$$\begin{cases} x = x_0 + tl \\ y = y_0 + tm \\ ax^2 + 2bxy + cy^2 + 2px + 2qy + r = 0 \end{cases}$$

we obtain a second degree equation in t , namely:

$$(al^2 + 2blm + cm^2)t^2 + 2[(al + bm)x_0 + (bl + cm)y_0 + pl + qm]t + ax_0^2 + 2bx_0y_0 + cy_0^2 + 2px_0 + 2qy_0 + r = 0.$$

Substituting the sum and product of the roots of this equation in (3), we obtain

$$t = \frac{ax_0^2 + 2bx_0y_0 + cy_0^2 + 2px_0 + 2qy_0 + r}{(al + bm)x_0 + (bl + cm)y_0 + pl + qm} \quad (4)$$

So the coordinates of Q satisfy:

$$\begin{cases} x - x_0 = tl \\ y - y_0 = tm, \end{cases} \quad (5)$$

where t is given by (4). Eliminating l and m in (5) we obtain

$$(ax_0 + by_0 + p)x + (bx_0 + cy_0 + q)y + px_0 + qy_0 + r = 0$$

which is the equation of the polar. For a matrix treatment of the subject see [1].

2 The Polar Conic

Let $A_1A_2A_3A_4A_5$ be a pentagon (Fig. 2). There is a unique conic section \mathcal{C} such that:

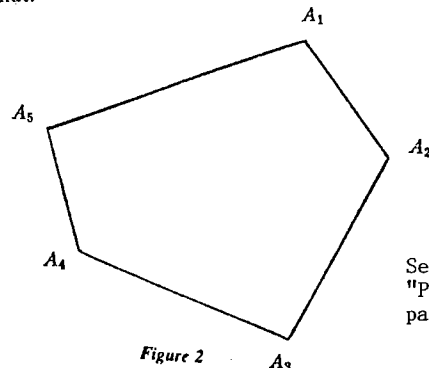
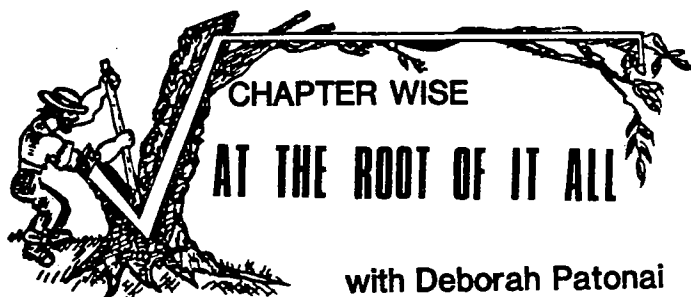
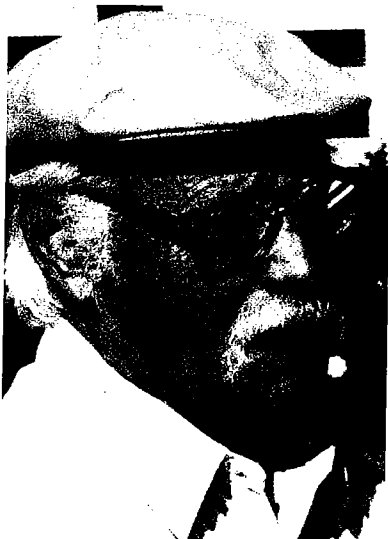


Figure 2

See "Polar Conic," page seven.



(Mathematical Log Activities Editor Deborah S. Patonai, perpetually youthful, lively, effervescent (she more than once has been mistaken for a student member!) at Seattle Convention met, and was rightly captivated by, a courtly, considerate "man of many [mathematical] interests," an articulate, quite fascinating "gentleman of the old school." She devotes her whole column to Dr. Ali R. Amir-Moéz. The Editor, who first met Ali in person at Seattle, quite echoes her sentiments, and acknowledges the Log Mathematics Editor as its staunchest supporter and ablest correspondent. --H.D.A.)



ALI AT SEATTLE

During Mu Alpha Theta's 17th National Convention this past Summer in Seattle [Debbie writes], a new face appeared among the many familiar ones. A tall, stately, distinguished gentleman with white hair and moustache was to be seen on lecture hall steps, rapidly and astutely dashing off pencil caricatures of both sponsors and admiring students. At other times, in the midst of a circle of admiring student members, he was to be viewed performing his distinctive magic with an improbable look of yellow cord. Who is this man—who, until now, has remained (for most) but a name beneath the titles of literally scores of carefully prepared "math content" articles in The Mathematical Log? This man is Ali Reza Amir-Moéz, friend, helper, and confidant of a succession of Editors, and, since 1982, Mathematics Editor of The Log (see December 1982 Log, p. 4).

Born in Teheran, Dr. Amir-Moéz was brought up in a family which highly valued education and culture. Although he failed in Algebra and Geometry (He has since authored works both subjects!--Ed.), he managed to get his high school diploma with the highest grade in his class. Later, after graduating from a military college, he attended University of Teheran, where he received his Bachelor of Arts in mathematics. As a young man he emigrated to the United States, furthering his formal education through the levels of mathematics M.A. and Ph.D. at University of California, Los Angeles. Since then, Dr. Amir-Moéz has served as instructor,

assistant professor, associate professor, and professor at schools across the country. He presently is Professor of Mathematics at Texas Tech University [P.O. Box 4319, Lubbock, TX 79409].

On the subject of mathematics, Dr. Amir-Moéz has written over 150 papers, articles, and books. His main field of expertise, he tells us, is "extreme properties of proper values and singular values of linear transformations and matrices." He is fluent in English, Persian, Spanish, and French, and his books and papers have appeared in all of these languages.

While mathematics, his life's work, is his first love, Ali is also interested--and talented--in many other fields: dramatics, literature, dance, painting, and working with gemstones to create remarkable jewelry. As an actor, Ali has performed in several experimental motion pictures, but none that have been released. In addition to his authorship of several plays, Ali has produced perhaps 75 books and features for children. His sketches and paintings have served to illustrate a number of works. Recently, he has developed a striking art medium, using crushed rocks (such as agate turquoise) in pictures to add dimension to details and to scenery. Ali's math-related artistic talents have been featured frequently in The Mathematical Log and Mathematical Tall Timbers. In fact, the graphics which head this column are an example of Ali's Log-"commissioned" good work.

This accomplished man has, we learned, been actively associated with Mu Alpha Theta since the 1960's. Josephine Andree and others convinced Ali to write a few articles for The Mathematical Log. Since these early contributions, Ali has prepared over 50 math articles for Log use (and for Chips compilations). For six years he has served ably as Log Mathematics Editor. (Don Allen underlines the importance of this continuing contribution: it is Ali who, with Texas Tech colleagues, will pass judgment on mathematical aspects of an adult or student submission, a vital service to the Log publishing team.) Ali's own distinctive contributions have become a regular, and educationally most important feature, of recent Log volumes.

I enjoyed my visit with you ... everyone at the Conference was charming and kind.

--Ali R. Amir-Moéz,
Communication to the Editor, August 1987.

At a particularly memorable "section meeting" at Seattle Convention, Ali Amir-Moéz presented one of his favorite math developments, "A Symbolic Approach to Cat's Cradle," a talk and demonstration based upon an algebraic notation for string figures. Members and sponsors fortunate enough to be ticketed for this small-group session were intrigued and fascinated by the mathematics and by the manual--and mental--dexterity of Ali's flexible yellow loops. During remaining days of the Convention, Dr. Amir-Moéz could be seen, seemingly everywhere, helping students perform the tricks and follow the lines of symbols. "If a talk brings even one person to ask questions or to write a letter requesting literature on the subject, one may boast that one has been effective," Ali believes. All the questions and letters that Dr. Amir-Moéz has received on this, his first Mu Alpha Theta presentation, indicate that he was effective--and engaging--indeed!

Throughout his many collections of sketches, caricatures, and photographs, Ali Amir-Moéz exhibits his artistic interests and talents. As a mathematician, he makes mathematics come alive. Involved with Mu Alpha Theta, he is as precious as the gems with which he so loves to work.

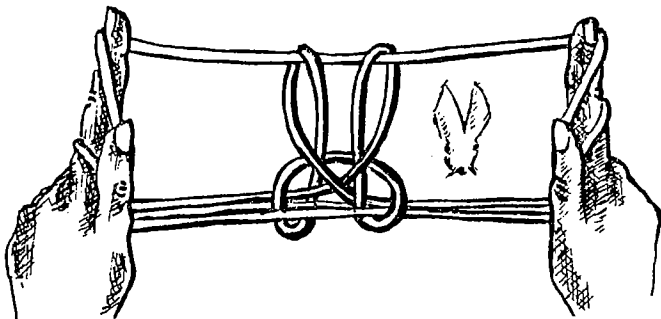
In closing, Dr. Amir-Moéz leaves us with the following quotation, from one of his works:

Whoever doesn't know
and doesn't know
that he doesn't know,
Forever remains in
perpetual ignorance,
and how!

Ali's String Rabbit

Here, by request, is the "algebra" for the ingenious "rabbit" string figure taught to all and sundry by the Log Mathematics Editor at Seattle Convention (see page 2).

$$a, B + x_1, B \cdot ex_1^*, k, A : h, A - (x_1, x_1^1, x_2, x_2^2, x_3), A \cdot e(x_3, x_2, x_2^2, x_1), k, A + x_1^1 - x_4, A \cdot ex_4, k, [\text{upper}B] \rightarrow A, A : n, E : h, (B, C, D, E,) + (x_4, x_3, x_2) - x_1, B \cdot ex_1, A : h, d.$$



REFERENCE:

Caroline Furness Jane, *String Figures and How to Make Them*, pp. 72-82, Dover, New York, NY (1962).

'Soap Bubble Math' Readings Shared

The mathematics, physical science, and visual and intellectual fascination of soap films, soap bubbles, and related "minimal surface" considerations, were strikingly revealed in a lively lecture-demonstration at a Seattle Convention general session. Speaking on "Bathematics: If Archimedes had only stayed in the tub, perhaps he would have discovered the wonderful world of bubbles," Dr. Millie Johnson, Western Washington University mathematics teacher, used soap film, transparent models, and an overhead to solve tricky, perhaps contrainuitive, problems of minimal connecting networks in the plane.

Dr. Johnson, at the Editor's invitation, provided The Mathematical Log with a list of selected references, suited to further study and from sources likely to be accessible to sponsors and student members. Some fairly easy reading, considerable practical experimentation, and the result could be an innovative program in applied mathematics for chapter presentation or an unusual, distinctly inexpensive, science fair demonstration.

Dr. Johnson may be contacted at Department of Mathematics, Western Washington University, Bellingham, WA 98225. Her suggested readings follow:

Almgren, F. J., Jr., and Taylor, J. E. "The Geometry of Soap Films and Soap Bubbles," Scientific American, July 1976, pp. 82-93.

Bader, William. "Problem Solving via Soap Bubbles," School Science and Mathematics, April 1975, pp. 343-53.

Boys, C. V. Soap Bubbles and the Forces Which Mould Them, Doubleday and Co., Inc., 1959.

_____. "The Soap Bubble," in James R. Newman, ed., The World of Mathematics, vol. 2, Simon and Schuster, 1956.

Courant, R., and Robbins, H. "Plateau's Problem," in James R. Newman, ed., The World of Mathematics, vol. 2, Simon and Schuster, 1956.

_____. and _____. What Is Mathematics?, Oxford University Press, 1941.

Ehrmann, Sister Rita M. "Minimal Surfaces Rediscovered," Mathematics Teacher, February 1976, pp. 146-52.

Hoffman, Dale T. "Smart Soap Bubbles Can Do Calculus," Mathematics Teacher, May 1979, pp. 377-85.

Hooper, J. "Math-Art," Omni, April 1986, pp. 88-91.

Isenberg, Cyril. "Problem Solving with Soap Films,"

Physics Teacher, January 1977, pp. 9-18.

_____. "Problem Solving with Soap Films, Part I," Physics Education, September 1975, pp. 452-56.

_____. "Problem Solving with Soap Films, Part II," Physics Education, November 1975, pp. 500-503.

_____. "Soap Films and Bubbles," Physics Education, July 1981, pp. 218-22.

_____. "The Soap Film: An Analogue Computer," American Scientist, September-October 1976, pp. 514-18.

Judson, H. F. The Search for Solutions, Holt, Rinehart, and Winston, 1980.

Lovett, David. "Soap Films, Phase Changes, and Catastrophes," Physics Education, January 1979, pp. 40-44.

Steen, L. A. "Solving the Great Bubble Mystery," Science News, 20 September 1975, pp. 186-87.

Stevens, Peter S. Patterns in Nature, Little, Brown, and Company, 1974.

Thompson, D'Arcy. On Growth and Form, abridged ed., J. T. Bonner, ed., Cambridge University Press, 1961.

Van Tassel, L. T. "A Homograph Based on Soap Bubble Geometry," School Science and Mathematics, December 1957, pp. 689-93.

Wahl, M. S. "A Permanent Soap Bubble Geometry," Arithmetic Teacher, April 1972, pp. 307-308.

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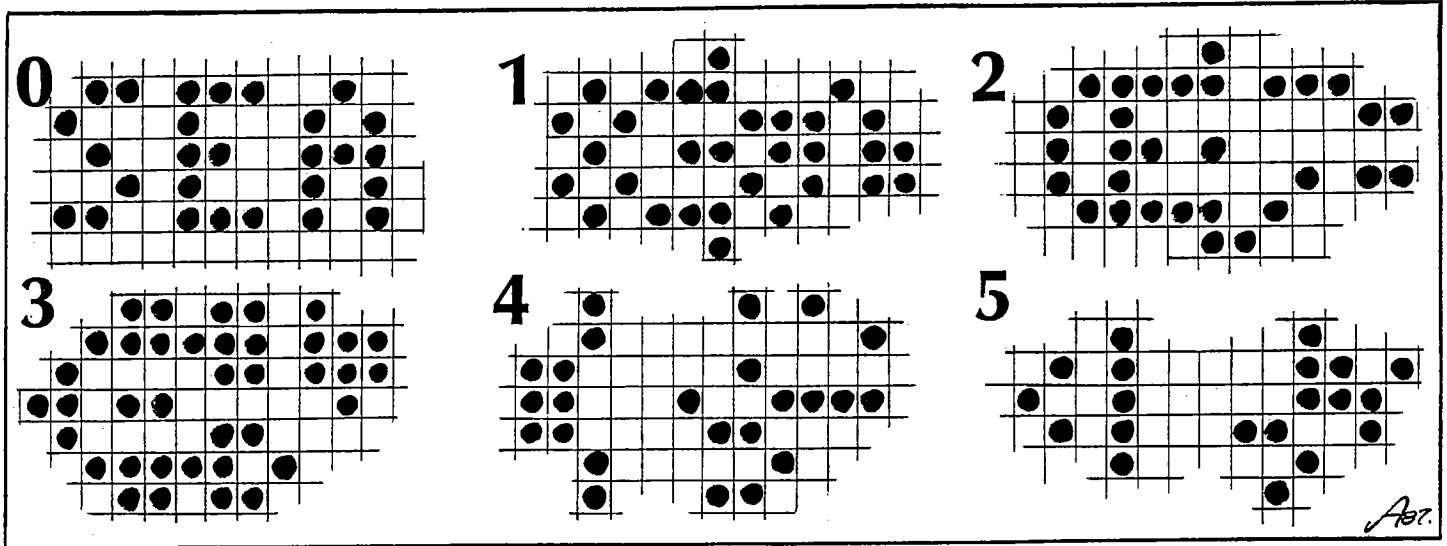
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STUDENT EXTENDS ACTIVITY

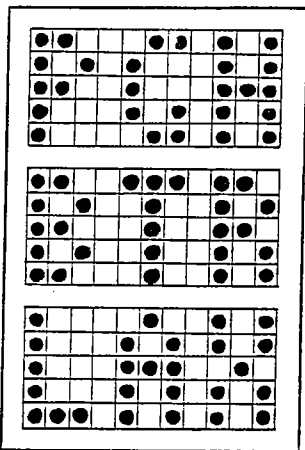


'LIFE' PATTERNS INTRIGUE TODD

The ultimate fate of SEATTLE? Ask Todd Belton, Mu Alpha Theta enthusiast from Baton Rouge, and he'll tell you: A "traffic lights" two-cycle of population 12, from Generation 17!

Todd, of course, is applying pencil, paper, computer, and agile mind to Conway's "Life" game (*Mathematical Log*, April, October 1985; *Scientific American*, October 1970), as played with the SEA travel designation of Seattle-Tacoma International Airport.

Your Log Editor distributed the 27-dots SEA rendering as a Seattle Convention challenge. Todd got a copy of the handout--and, characteristically, ran with it all the way!



"Life" rules stipulate, as Todd well knows, that a dot in a life pattern "survives"--persists into the next "generation"--iff two or three of the eight "cells" neighboring its cell are "occupied": otherwise it "dies," of isolation (less than two occupied neighboring cells) or of overcrowding (four or more occupied neighboring cells). In addition, new cells are "born" in empty cells which have had exactly three occupied neighboring cells.

Todd treated the 27-dot SEA as Generation 0 (above), and logged 6 deaths from isolation, 3 deaths from overcrowding, and 13 births, to yield the 31-dot pattern which we have recorded as Generation 1. He follows logically with

the 30 dots of Generation 2, 38 dots of Generation 3, 24 dots of Generation 4, and 20 dots of Generation 5. Todd notes that the dot "population" is reduced to 4 by Generation 8, then grows to 20 by Generation 16 before stabilizing at 12 for the "traffic lights" cycling.

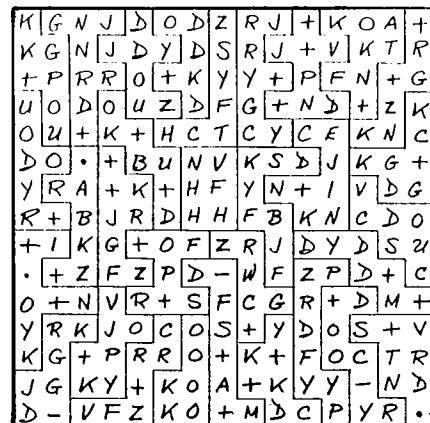
Todd, adept at pencil-and-paper Life, used his computer program for the SEA and related investigations.

Todd is aware that major civilian airports, worldwide, have such three-letter designations--used for tagging luggage, as one instance. He writes: "I got interested in this idea of Life formations from airport codes ... airport codes have always fascinated me ... and did a few more." His results for BTR (Baton Rouge, his home town), PGH (Pittsburgh, where he has studied), and LAX (Los Angeles International) he communicated in detail.

As-represented in the lower diagram, BTR (Baton Rouge) "stabilizes" at Generation 31, Todd reports. LAX (Los Angeles) stabilizes at Generation 43. PGH (Pittsburgh) is not to be taken on lightly, however. Todd reports that it yields "a pattern of 15 circular stable forms, 4 blocks, and 9 blinkers" ... but only at Generation 945!

Life, of course, makes for a fascinating Mu Alpha Theta chapter program, with great follow-up possibilities. Good literature on the subjects now abounds.

Further, Todd is an outstanding correspondent, and no doubt would welcome dialogue on any discoveries. A recent graduate of McKinley High School, his home address is: 11622 Pamela, Baton Rouge, LA 70815.



CHECKERBOARD REASSEMBLED--Here's one reconstruction of our 31-piece "Days in August" checkerboard cipher challenge, as featured at Seattle (December Log). The 15 x 15 checkerboard now carries an enciphered, math-related sentiment. The next phase: retrieve the "message." Any solutions?

766-3844, 648-6879...

'TELEPHONE DIAL' MATH PROBLEMS DISTINCTIVE LOG CHALLENGE

by Don Allen

Telephone dials, and now their "pushbutton" counterparts, associate decimal digits with alphabet letters in a manner that all have observed but not all have reflected upon. How many recall how--and when--"exchange" names were dialed (the initial two letters), followed by four (later five) digits, as easier to say and to remember than the all-digit "phone numbers" we have today? Thus, Harbour 2065 (our boyhood "number" in Montreal) was simpler to use and recall than the 422065 to which it was equivalent. The letters somehow persist on North American telephone dials, with little use except as an occasional charity or merchandising memory aid. Thus, in present-day Montreal, you dial AUTOBUS (which really is 288-6287) to learn municipal bus and subway routings and schedules for this city.

The letters on telephone dials can, however, be the basis for a range of fun activities. Our favorite is to "cast" a riddle or problem in phone numbers, rendering deciphering a multiple-letter cryptographic challenge--in which, it turns out, the human mind can do quite remarkable things. (See "Telephone Dial Correspondence Conceals Attractive Old Riddle in Jingle Form," *Mathematical Log*, 26:1 (Fall 1981), p. 4; reprinted in Josephine P. Andree, ed., *Mathematics to Play and Ponder*, 1984, p. 60.) What follows are two new, previously unpublished, telephone dial challenges, based on word problems (in verse) from early nineteenth century Arithmetics. Convert digits back to letters, then read and solve each problem. See who in your chapter can do it first.

To demonstrate (sketch a phone dial!), MU ALPHA THETA CHALLENGE is enciphered

MU ALPHA THETA CHALLENGE
68 25742 84382 242553643,

or, as seven-digit "phone numbers," 682-5742 843-8224 255-3643. Elementary! The reverse process, however, might, at first thought, seem difficult, even impossible. Thus, the three "numbers," 835-3746 633-4256 377-2437, given the three-to-one letter-digit association, ought to permit some three to the 21st (billions!) interpretations, but setting up the matching,

835374663342563772437
TDJDPGMMDDGJAMDPPAGDP
UEKERHNNNEEHBKNERRBHER
VFLFSIOOFFICLOFSSCIFS,

causes the one "correct" one, TELEPHONE DIAL MESSAGES, to all but leap out. Such, we suggest, is the power of the mind to make associations.

Try, now, these two new "telephone dial encipherments" of problem classics. For each, the two-fold challenge is to decipher and to solve. Here is raw material for a lively--and different--chapter competition or collaboration.

MARITAL MATTERS

943-6347 788-4362 774-2435 668-9278 433-2389 336-6994
332-6363 692-4392 786-8428 636-9274 332-7847 338-4637
847-3386 847-3328 866-9943 683-6263 425-3836 932-7793
626-2639 433-4283 233-6437 243-8664 633-9228 592-3277
273-4448 478-6749 833-6669 835-5477 293-7669 428-4837
243-9428 937-3687 243-7943 693-9331

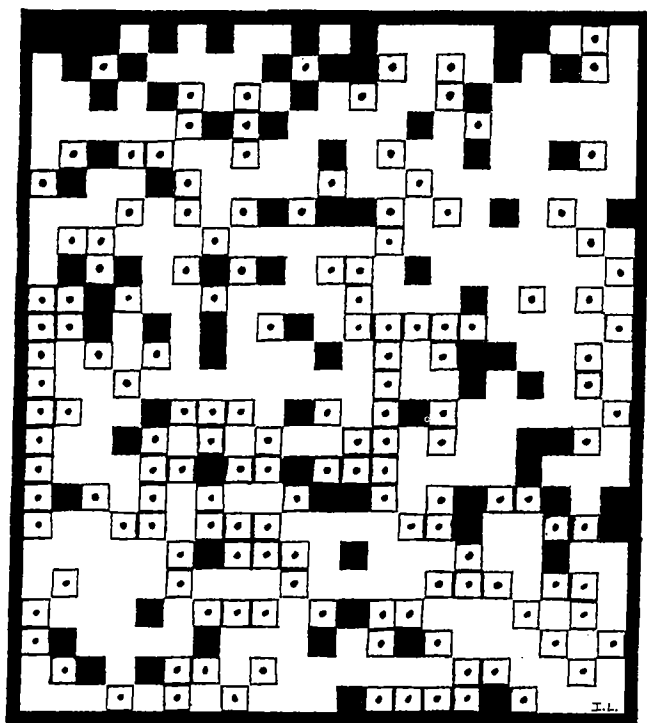
GEOMETRIC DOWRIES

243-6853 626-8963 284-4837 742-3263 268-4937 383-7932
478-6322 443-4283 274-3236 352-6366 376-8638 436-8437
759-2732 836-7893 245-3784 322-7358 783-2247 432-3487
825-8342 384-3365 527-7842 836-2667 277-3332 243-6732
243-9228 597-2434 327-6772 365-5272 326-4624 274-8478
379-6327 946-4238 432-3883 776-7846 684-2842 384-3768
636-7759 273-1101

The problems derive from *Willetts's Mental and Practical Arithmetic* (Poughkeepsie, 1849) and Tobias Ostrander, *The Elements of Numbers, or Easy Instructor* (2nd ed., Canandaigua, 1823). The solution to the first problem, as provided in the text:

849-2439 436-6277 433-6878 428-3233 658-7836 789-3483
849-9433 734-3833 611-0101.

* * *



MAKE IT VISUAL, suggests Isaac Lin of Montréal, summer school student of the Editor, who here "pictures" a 1609 conjecture of DeBouvelles on a distinctive checkerboard array. DeBouvelles was investigating what we would call twin primes, pairs of consecutive odd integers both of which are prime (as 11 and 13, or 59 and 61). He noted that such prime pairs (after 3, 5) all are numbers one less than and one more than a multiple of 6. Thus, such pairs may be written $6k - 1$, $6k + 1$, for suitable k . DeBouvelles' conjecture was that, for all $k > 0$, one or both of the numbers given by $6k - 1$, $6k + 1$ must be prime. As Isaac quickly noted, the conjecture is false, for $k = 20$ (which yields composites $119 = 7 \times 17$ and $121 = 11 \times 11$), and [infinitely] many other times. Isaac's 21×24 grid pictures what happens through $k = 504$. The cells are numbered by rows, 1-21, 22-42, etc., the numbers corresponding to successive k values. Where $6k - 1$ and $6k + 1$ both are prime (as for 1, 2, 3, 5, 7), Isaac renders the cell dark. Where one value is prime, the other composite (in either order), the cell is light. "Dotted" cells correspond to k values yielding $6k - 1$, $6k + 1$ both composite--values for which DeBouvelles' conjecture clearly fails--an interesting "pattern" in themselves: note the marked tendency to "line up." Isaac found his design, initially rendered in color, "quite pleasing to the eye," and has suggested its reinterpretation as a spiral array or on a triangular or hexagonal grid. The prime numbers are rich in possibilities for such conjectures--and such visualizations.

