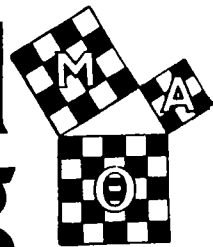
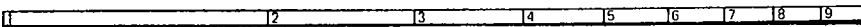


The Mathematical Log



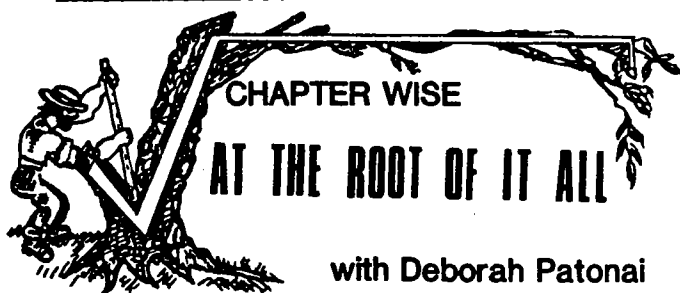
Volume 32, Number 4 December 1988



Closing Log entry for 1988: $1988 = 1^2 + 9^2 + 8^2 + 8^2 + 1^3 + 9^3 + 8^3 + 8^3 + \sqrt{1.9.8.8}$



A POSTAGE STAMP HONORING MU ALPHA THETA? No, not really--although, reflecting on the unique contribution of the national organization to the lives of hundreds of thousands of young Americans, the idea (at some future date) really might not be that far-fetched! The stamps are Greek, of thirty years ago, and mark the 2500th anniversary of the founding, by Pythagoras, of the first school of philosophy, on the island of Samos, shown on the map stamp. Pythagoras himself is depicted as on a contemporary coin. The striking resemblance of the most famous of mathematical diagrams to the Mu Alpha Theta emblem above, is interesting. This Log issue presents a computer program for reproducing the Mu Alpha Theta emblem (p. 7).

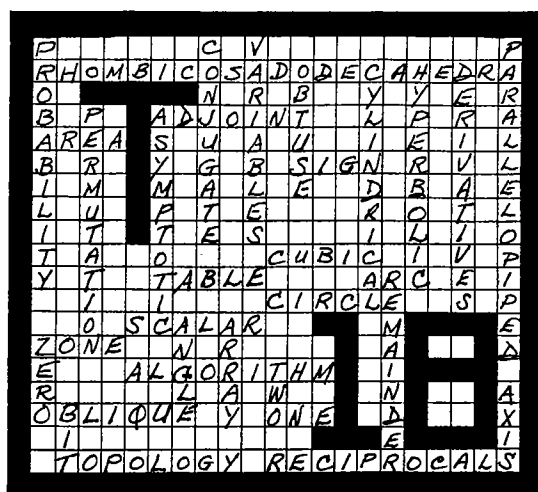


Mathematical Log Activities Editor Deborah S. Patonai welcomes your help in reporting each issue on the people, the ideas, and the happenings that are, in total, Mu Alpha Theta. Send news and views from your school and your Mu Alpha Theta chapter to Miss Patonai at Saint Vincent-Saint Mary High School, 15 N. Maple St., Akron, OH 44303.

Miss Patonai writes:

The word "challenge" means "anything, as a difficult task, that calls for special effort." In order to make their chapter significant, most Mu Alpha Theta members and sponsors must face numerous challenges. Several individuals have proven exceptional--outstanding--in meeting these challenges. To recognize and honor individuals from this select group, the national organization of Mu Alpha Theta has introduced two prestigious awards: the Kalin Award, given to an outstanding senior graduate in attendance at the national convention, and the Huneke Award, presented to an outstanding adult sponsor. (See the December 1986 Log for background information on both awards.) At this time, $\sqrt{\text{At the Root of It All}}$ is proud to recognize the 1988 recipient of the Huneke Award, Sister M. Scholastica

(Continued on pages two, eight)



PHASE I WINNER! The "T" for Tennessee and "18" to salute Mu Alpha Theta's 18th National Convention in Knoxville recall your Mathematical Log "Math-Pack" Fall Contest (October Log, page 3), and the imposing 1030 points claimed for this winning Phase I entry identifies the at-Convention efforts of William Fisher of Alamo Heights High School, San Antonio, TX. William receives the first Logmaster patch of the new season for his efforts, and the contest now "goes national" with Log readers and friends coast to coast (and abroad) invited to better William's top score. Full rules and point values for scoring are in October's Log issue.

Future Teacher Award To Arkansas Member

Proposed by Governing Council in 1986 and named for the organization's distinguished co-founder, Mu Alpha Theta's Richard V. Andree Award is being presented annually to a deserving member or former member intending to embark on a career as mathematics teacher. This year's winner, presented with her cash award and plaque at Knoxville convention, was Teresa Britt of Northside High School, Fort Smith, AR. Teresa, whose high school sponsor was Mrs. Kathy Rogers, plans to attend Westark Community College, then transfer to Arkansas State University, majoring in Mathematics Education. Teresa, commenting to The Mathematical Log, has stressed that she had "never dreamed that the Award would bring [me] such [recognition]," noting that "those at school and everyone who read about the award in the newspaper congratulated me."



Along with the to-be-expected documentation, Andree Award candidates are asked to comment with some care on their career choice. Like most such essays, Teresa's submission, as received by National Office, contains much food for thought. Its text follows:

I want to be a mathematics teacher for three reasons.

First, I do well in my math classes, because I have no trouble understanding my assignments. Second, I like to teach people new things. I enjoy helping people to understand what they are doing. If people do not understand what they are doing, then they will never learn, and I know how much I hate not being able to understand an assignment. Finally, I want to keep in touch with the times. I do not want to lose the knowledge that I have already achieved. Becoming a teacher is the best way to keep that knowledge. I will always be going back to school, because there will be changes as the years pass. I feel that if I do not become a teacher then I have wasted twelve years worth of knowledge. I want to be able to relate to the people of the time. For the reasons that I have just mentioned, I want to be a mathematics teacher.

AT THE ROOT OF IT ALL

... FROM PAGE ONE

Friesenhahn, and of the Kalin Award, student Rajit Jain. Both have accepted the Mu Alpha Theta challenge.

The deserving winner of this year's Huneke Distinguished Sponsor Award, Sister Scholastica, is mathematics teacher and Mu Alpha Theta sponsor at Blessed Sacrament Academy, San Antonio, TX. Involved with Mu Alpha Theta since the early 1970's, Sister Scholastica has thrived on the challenge of creating and maintaining an exemplary chapter. Over the intervening years, her chapter has attended countless state and local gatherings and contests, as well as every National convention. Often winning mathematics competitions and state and national offices, Sister Scholastica's students have very well represented their school.

Being the motivating force for the various projects initiated by the club, Sister Scholastica keeps herself and her students extremely busy. One of her major projects is fundraising, by which she helps her students raise the money for convention attendance and participation. In fact, Sister has become something of a fundraising expert.

This money raised allows many of her students to go on trips which they otherwise could not afford.

Not only does Sister Scholastica involve herself with her school's Mu Alpha Theta chapter, she also is active on local, state, and national levels. Some years ago, she was elected, and served with distinction, as Mu Alpha Theta national governor from the then central region. She also has been active at NCTM regional and national meetings. Even on this level, she has effectively involved her students, especially as student aids. Her Mu Alpha Theta members go about their duties at these important meetings...creating a warm and friendly atmosphere.

The major challenge that Sister Scholastica faces, as teacher and sponsor, is bringing to her students a greater appreciation for, and deeper appreciation of, mathematics. Encouraging her students to attend every possible mathematics meeting and competition, she assures these students opportunities to develop and display math abilities, to hone leadership skills, and to take on responsibilities. In fact, Sister tries to extend these opportunities to as many students as possible. By creating associate memberships for underclassmen not yet qualified for Mu Alpha Theta membership, she "involves" still more students.

The Principal of Blessed Sacrament Academy, Sister Alberta Novosad, praises Sister Scholastica: "It is spon-

(Concluded on page eight)

The Mathematical Log

ISSN 0025-5580

Volume 32, Number 4

December 1988

The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Mu Alpha Theta, founded in 1957 by Richard and Josephine Andree, is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). The Mathematical Log is published quarterly, in February, April, October, and December. Correspondence may be directed to specific editors or to Mu Alpha Theta National Office, 601 Elm Ave., Rm. 423, Norman, OK 73019. Contents copyright © 1988 by Mu Alpha Theta.

MU ALPHA THETA

President: Pamela J. Drummond, Department of Mathematics, Walton High School, 1590 Bill Murdock Rd., Marietta, GA 30062.

President-Elect: Dr. John Kenelly, National Science Foundation, 1800 G St. NW, Rm. 638, Washington, DC 20550.

Secretary-Treasurer: Dr. Thomas J. Hill, University of Oklahoma, 601 Elm Ave., Rm. 423, Norman, OK 73019.

Governing Board: Jeanne Nelson, Kamehameha Schools, Kapalama Heights, Honolulu, HI; Lynn Britton, John Jay High School, 7511 Marbach Rd., San Antonio, TX 78227; Gwenda Snoddy, Austin High School, 1626 Danville Rd. SW, Decatur, AL 35601; Annabelle Robinson, Lebanon High School, Harding Dr., Lebanon, TN 37087.

M.A.A. Representative: Katherine P. Layton, Beverly Hills High School, Beverly Hills, CA 90212.

N.C.T.M. Representative: Barbara Stott, Riverdale High School, 240 Riverdale Dr., Jefferson, LA 70121.

THE MATHEMATICAL LOG

Editor-in-Chief: Dr. H. Don Allen, 6150 Bienville Ave., Brossard, Québec, Canada J4Z 1W8.

Activities Editor: Deborah S. Patonai, Saint Vincent-Saint Mary High School, 15 N. Maple St., Akron, OH 44303.

Mathematics Editor: Dr. Ali R. Amir-Moéz. Please address correspondence to Mu Alpha Theta National Office.

19th NATIONAL CONVENTION, TAMPA, FL, AUGUST 3-8, 1989

Sponsoring chapters: King High School, Tampa, FL; Berkeley Prep School, Tampa, FL; Plant City High School, Plant City, FL. Registration particulars: Dave Steele, Plant City High School, One Raider Pl., Plant City, FL 33566 (school: 813 754-1541; residence: 754-8306).

1855 Arithmetic

Knoxville Textbook 'Find'

Rich in Story Problems

By Don Allen

"Word problems" in old school mathematics textbooks long have had, for us, a two-fold fascination. "Stories" of improbable agreements, contests, and undertakings—themselves unconscious reflections on the "useful" and "relevant" of other times and places—can offer fascinating insights into ways of life and thought. Equally, the nature of, and possible approaches to, the more distinctive of such "story problems" can reveal the manners in which associated mathematical topics were considered, applied, extended, and tested.

Accordingly, chancing upon an eighteenth or nineteenth century school or college mathematics textbook with which we are not already familiar can be a highlight of summer travel, and pleasant surprises do await in university libraries and antiquarian bookstores far from home. We chanced upon a remarkably well preserved copy of John F. Stoddard, A.M., The American Philosophical Arithmetic (New York, 1855) at The Incurable Collector, "Books ... Unique, Antique," in Knoxville while attending the Mu Alpha Theta national convention at University of Tennessee in August. This comprehensive compilation was "designed for the use of advanced classes in schools and academies," according to its title page: Stoddard was president of University of Northern Pennsylvania.

From initial consideration of numeration ("French method," through hundreds of decillions) and "fundamental rules," the old text quickly proceeded to "more advanced principles of the science of numbers, and their applications to practical purposes": denominate numbers, division of duodecimals, reduction of currencies, mercantile calculations, alligation medial and alternate, involu-tion and evolution (arithmetic powers and roots), mensurations of surfaces and solids, and "philosophical problems" (what we would recognize as physics). We hastened, however, to the "miscellaneous questions" final chapter,

Early math texts were strong on "rules": to be committed to memory, then routinely "applied." Stoddard offers this rule for determining the solidity [volume] of a [regular] dodecahedron, given its side [edge]:

Add 47 to $21\sqrt{5}$, and divide this sum by 40; then multiply the square root of this quotient by 5 times the cube of the side [edge].

a 143-problem compendium of the kind of sundry, usually non-routine word problems that the nineteenth-century "back of the book" browser might have come to expect.

Early questions from such problem sets can be both interesting and instructive ... and, we assert, good fun. Not infrequently the first challenge will be to "understand" the problem, to grasp its intent and to choose an attractive, effective approach to solution. In this spirit, we've carefully selected, to share here, a dozen of what we feel to have been Stoddard's best. Try them, some or all, enjoy and share the challenge ... and, we suggest, try to play by Stoddard's implicit rules.

No computer generated solutions, no calculators, no logarithms ... indeed, no Algebra: Arithmetic meant Arithmetic in the nineteenth century classroom. (Note, too, the degree to which Stoddard's was a fraction, and not a decimal world.)

The twelve questions follow. Note that the first two call for rather elaborate manipulation of numbers. Nineteenth century pupils would have known that such questions either culminated in "nice" answers (for example, 1), or rather pointedly did not! Stoddard's did not. Subsequent questions cover a delightful range of topics and skills, from business math through mensuration, stressing rather involved considerations of ratio and proportion. One problem seems to call for the Chinese Remainder Theorem (see Underwood Dudley, Elementary Number Theory, 2d ed.), though Stoddard must have had a more rudimentary approach in mind.

Notes on problem interpretation, answers obtained, instructive approaches to solution, will be welcome for any or all of the following questions, for possible sharing with Mathematical Log readers.

I. What Result?

What will be the result, if from the sum of $3, \frac{3}{4}, 3\frac{1}{2}, \frac{3}{4}$ of 3, $3\frac{1}{2}$ of $3\frac{1}{4}$, $\frac{5}{7}$ of $\frac{3\frac{1}{2}}{2\frac{1}{2}}$, we subtract the sum of $\frac{1}{3}, \frac{1}{3}$ of $\frac{1}{3}$, $\frac{1}{4}$ of $3\frac{1}{2}$; $\frac{2\frac{1}{2}}{3}$ of 5, $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ of $\frac{3\frac{1}{3}}{2\frac{1}{2}}$, and $3\frac{1}{8}$; multiply

this difference by the greatest common divisor of 315 and 405; divide this product by the least common multiple of 6, 9, and 24; reduce this quotient to its lowest terms; add $\frac{1}{3}$ of $\frac{3}{4}$ to the result; multiply $\frac{3}{4}$ of this sum by $2\frac{1}{2}$; and divide the product by $\frac{1}{2}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ of $\frac{32}{45}$ of $\frac{2\frac{1}{2}}{2\frac{1}{3}}$?

II.

Sum, Difference, Product, and Quotient

Make the sum, difference, product, and quotient of 15 and 45 the numerators of fractions which shall have 75, 40, 750, and 60 for denominators; reduce them to equivalent fractions having a common denominator; subtract the sum of the last two fractions from the sum of the first two; multiply the difference by the first fraction; divide the product by the greatest common divisor of the numerators; multiply the quotient by the least common multiple of the denominators; add the first fraction reduced to a decimal to this quotient; subtract the second fraction reduced to a decimal from this sum; multiply this remainder by the third fraction reduced to a decimal; divide this product by the fourth reduced to a decimal; then reduce the quotient to a vulgar fraction. What is the result?

(See "1855 Arithmetic," page four)

1855 Arithmetic

. . . CONTINUED FROM PAGE THREE

III.

The Least Possible Number

Find the least possible whole number which being divided by 28, shall leave 19 for a remainder; and being divided by 19, shall leave 15 for a remainder; and being divided by 15, shall leave 11 for a remainder?

IV.

To Go to New Haven

Two men in New York hired a carriage for \$25, to go to New Haven, a distance of 72 miles, and return, with the privilege of taking in three more persons. Having gone 20 miles, they take in A; at New Haven they take in B; and when within 30 miles of New York they take in C. How much in equity ought each man to pay?

1. "Lasting impressions, Knoxville, 1988." APEX DX.

VO UXBEOXWK, VEXIDVUUURK, GQRUUBHR BY AGZ *XLTLYKZ KLY. TURKZ ABEEK, WRRH TURRY, MURXZARW BY AXPR. MBEW QBYRK BY FXCRKZBI SURR=SGUFK. ZGMURBYT ZALYWRUARXWK; CGEZK GS EBTZYBYT; JLBID, BYZRYKR, WGNHYGLUK; ZARY, TXBY, KLY.

2. "Stroll at sunset, East Tennessee, 1988." APEX DX.

PHFG ATT IVYL FGDSSG, NYONKVQF, TAFFQ WDAAE KSLZF YZQRXYX KVQ BVFG VOSZ GDSSF, JYLSF, FNDHWF AT *ELAMJYRRS'F *GQFAL *BVDE. FGVDF DSVRYGQ: "VJAYZ WAZQ XALGVXG. FGDSVI TVYRF WYARAOYXVR FGVLVZVDF," FGVGS NSVRGN VHGNDYQG XVII-GYALF.
--Enciphered insights... Good solving!

V.

How Many Cannon Balls?

How many cannon balls, 8 inches in diameter, can be put into a cubical vessel, 2 feet on a side; and how many gallons of wine will it contain after it is filled with balls, allowing the balls to be hollow, the hollow being 4 inches in diameter, and the opening leading to it, to contain $1\frac{1}{2}$ solid inches?

VI.

How Many Soldiers?

A general drew up his regiment in the form of a square and had 94 men remaining; soon after a detachment of 485 men more joined him, whereby he was enabled to increase the side of the square by 3 men. How many soldiers had he at first?

VII.

Butter, Strawberries and Eggs

A market-woman carried some butter, strawberries and eggs to market; she sold her butter, at 25 cents a pound; her strawberries, at 20 cents a quart; and her eggs, at 15 cents a dozen; the whole amounted to \$7.65. The number of pounds of butter equalled the number of dozens of eggs increased by the number of quarts of strawberries; and the number of pounds of butter increased by the number of quarts of strawberries, or the number of dozens of eggs, would equal 3 times as much as the remaining number. What was the quantity of each article?

VIII.

Providing All Work

A, B, C, and D agree to a certain piece of work, for \$945; A, B, and C can perform the work in 84 days; A, B, and D, in 72 days; A, C, and D, in 63 days; and B, C, and D, in 56 days. How much money should each receive, providing they all work until the work is complete?

IX.

How Much Had Each?

A, B, C, and D play cards on this condition: that he who loses shall give to all the others as much as they already

have. First A lost, then B, then C, and then D. When they began to play they had \$162, \$82, \$42, and \$22, respectively; how much had each at the end of the fourth game? Suppose, when they had all lost in turn, that each had the same sum of money, \$96; how much had each when they commenced to play?

X.

There Is an Island

There is an island $26\frac{2}{3}$ miles in circumference, and three men, A, B, and C, start from the same point, and travel in the same direction around it; A goes $2\frac{1}{2}$ miles an hour; B goes $8\frac{1}{3}$ miles an hour; and C goes $9\frac{2}{7}$ miles an hour. In what time will they all first be together; and when will they all be together at the place from which they started?

TRY SOMETHING SIMPLER

Our twelve Stoddard "miscellaneous questions" have been for their intrinsic challenge and their call for originality and resourcefulness in solution. For those who might prefer beginning with something more elementary, the following short problems also derive from the Stoddard work of 1855. An arithmetic solution would have been expected. "Trial and error" methods would have been unlikely to have been accepted, we feel.

1. Henry is 18 years old, and Harvey is 14; how many years since was Henry twice as old as Harvey?

2. A farmer gave a certain number of laborers, \$14 apiece, if he had given them \$19 apiece it would have taken \$125 more. How many laborers were there?

3. A man bought a horse for \$102, which was $\frac{3}{7}$ of twice as much as he sold it for, lacking \$2. How much did he gain by the bargain?

4. A person being asked his age, replied, that if his age were increased by its $\frac{2}{5}$, its $\frac{5}{6}$, and $25\frac{1}{2}$ years more, the sum would equal $3\frac{1}{2}$ times his age. What was his age?

5. A man being asked the hour of the day, replied, that $\frac{3}{5}$ of the time past noon equalled $\frac{3}{4}$ of the time from now to midnight. What was the time?

And, an eighteenth century "type problem" included by Stoddard in his end-of-book set:

6. There is a fish whose head is 18 inches long, and whose tail is as long as its head + $\frac{2}{3}$ of the length of its body, and whose body is as long as its head and tail both. What is the length of the fish?

XI.

20, 15, and 10 Dollar Bills

A gentleman has in one bank a certain number of 20, 15, and 10 dollar bills; in another a certain number of 5, and $2\frac{1}{2}$ dollar gold coins. The number of bills and coins in both banks equal[s] 3224. How many of each has he, providing $\frac{3}{4}$ of the number of 20 dollar bills equal $\frac{2}{3}$ of the number of 15 dollar bills, $\frac{3}{4}$ of the number of 15 dollar bills equal $\frac{2}{3}$ of the number of 10 dollar bills, and $\frac{3}{4}$ of the number of 5 dollar gold coins are 48 more than $\frac{2}{3}$ the number of $2\frac{1}{2}$ dollar coins; also, that $\frac{5}{7}$ of the number of bills equal $\frac{5}{6}$ of the number of coins; and what amount of money has he in both banks?

XII.

Grazing Oxen

A and B paid \$90 for 12 acres of pasture for 8 weeks, with an understanding that B should have the grass that was then on the field; and A, what grew during the time they were grazing. How many oxen according to the above understanding can each turn into the pasture, and how much should each pay, providing 4 acres of pasture, together with what grew during the time they were grazing, will keep 12 oxen six weeks; and in a similar manner, 5 acres will keep 35 oxen two weeks?

'COMPASSES ONLY' RESTRICTION ADDS GEOMETRIC CHALLENGE

By Ali R. Amir-Moéz

Geometry and geometrical constructions have been the pastime of many people in all walks of life. This interest has become so great that people have challenged each other to construct geometrical figures with only one instrument, such as a straightedge or a pair of compasses.

It is said that Napoleon Bonaparte challenged his engineers to divide a circle into four congruent parts with only a pair of compasses. This problem has been solved by Lorenzo Mascheroni. Furthermore, Mascheroni and Mohr, two mathematicians of the eighteenth century, independently proved that every construction which can be done by straightedge and compasses can be done using a pair of compasses only—except, of course, the actual drawing of a straight line.

1 Napoleon's Problem

Draw a circle with center O and mark a point A on it (Fig. 1). Without changing the radius, put the needle of the compasses on A and mark the point P . Now put the needle of the compasses on P and mark the point Q . Finally, put the needle of the compasses on Q and mark the point B . The line segment AB is a diameter of the circle and passes through the center of the circle. It is clear that the two points A and B divide the circle into two congruent parts.

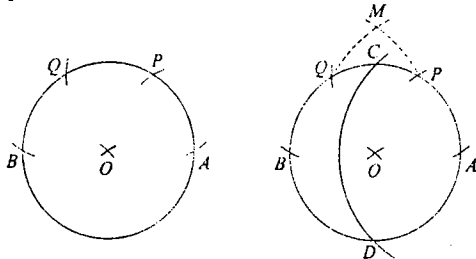


Fig. 1

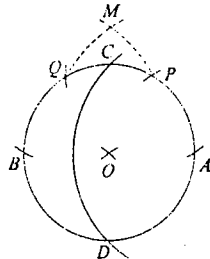


Fig. 2

Now let us repeat the diagram and make (Fig. 2). Put the needle of the compasses at A and with radius AQ draw an arc above the circle. Put the needle of the compasses at B and with radius BP , which is congruent to AQ , draw another arc above the circle. These two arcs intersect at the point M . Now choose OM for the radius and with center A draw an arc so that the arc intersects the circle at points C and D . The points $A, C, B,$ and D divide the circle into four congruent parts. The proof is left to the reader.

2 Geometry with a Pair of Compasses Only

If we examine any geometric construction, we find that the construction consists entirely of finding the positions of certain points. It is clear that the points of intersection of two circles can be obtained with only a pair of compasses. We shall now say a few words about the points of intersection of a circle and a straight line and the point of intersection of two straight lines.

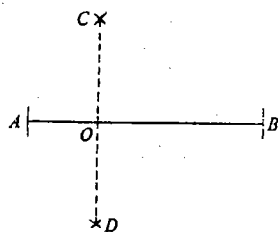


Fig. 3

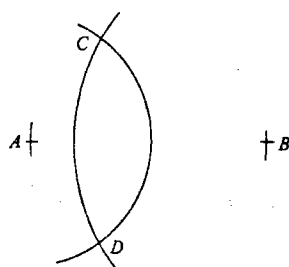


Fig. 4

3 Symmetry

Suppose a line AB and a point C which is not in the line are given (Fig. 3). Construct the line through C which is perpendicular to the line AB . Let O be the point of intersection of the line AB and the perpendicular from C . Now extend CO to a point D so that $CO=OD$. The point D is called the *symmetrical* of C with respect to the line AB . Sometimes D is called the *mirror reflection* of C with respect to the line AB .

Now, suppose two points A and B are given (Fig. 4). We want to find the *symmetrical* of a given point C with respect to the line AB . We can do this using only a pair of compasses. Put the needle of the compasses at A and, with radius AC , draw an arc. Now put the needle of the compasses at B and, with radius BC , draw another arc which will intersect the previous arc at C and D . The point D is the mirror reflection, or *symmetrical* of C with respect to the line AB .

4 Points of Intersection of a Circle and a Straight Line

Suppose a circle with center O and two points A and B of a straight line are given (Fig. 5). First we find the *symmetrical* of O with respect to the line AB . To do this, we repeat the procedure described in Section 3. With center A and radius AO draw an arc. With center B and radius BO draw another arc. These arcs intersect at a point P different from O . The point P is the *symmetrical* of O with respect to the line AB . Now draw a circle with center P and the radius the same as the one of the given circle. This

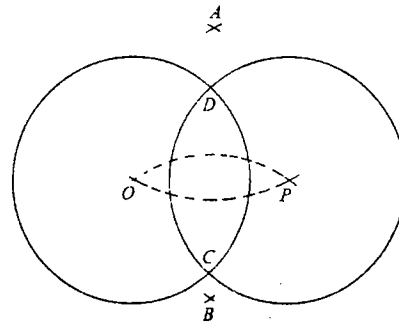


Fig. 5

new circle intersects the first circle at two points C and D . These points are the points where the line AB intersects the circle with center O . We can check the result by using a straightedge.

If the circles with centers O and P do not intersect, then the line AB does not intersect the given circle. If the line AB passes through O , the center of the circle, this method will not work since the mirror reflection would be the same point, O . The reader may study the construction in this case.

5 The Point of Intersection of Two Straight Lines

This construction is quite easy, but a little long. Since only one line passes through two distinct points, we can think of a pair of points as representing a line. Suppose the points A and B represent one straight line and the points C and D represent another straight line (Fig. 6). We want to find the point of intersection of these two lines. Remember, since these constructions are with compasses only, we cannot draw lines AB and CD .

(See "Geometry with Compasses Only," page six)

