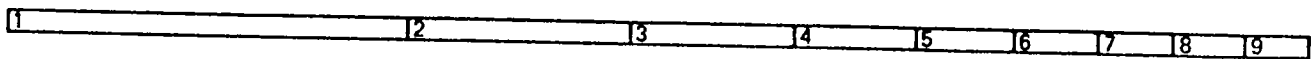


the mathematical LOG



VOLUME 31, NUMBER 1

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Tetrahedral Properties Triangular Analogues

By Ali R. Amir-Moéz
Mathematics Editor

A right triangle has many interesting properties which may be generalized to a right tetrahedron, that is, the one which has three right angles at one of its vertices.

In this article we shall study some of these properties and suggest others. We shall only give synthetic proofs, and purposely, avoid vector algebra and analytics geometry. We also try to give clear diagrams with outlines of proofs.

1 The Median

Let ABC be a right triangle, where A is the vertex of the right angle (Fig. 1). It is well-known that the median corresponding to the hypotenuse is half of the hypotenuse, i.e., $AM = \frac{1}{2}BC$, where M is the midpoint of BC . Let $AB = b$, $AC = c$, $AM = m$. Then one can show that

$$m^2 = \frac{1}{4}(b^2 + c^2). \quad (1)$$

We omit the details.

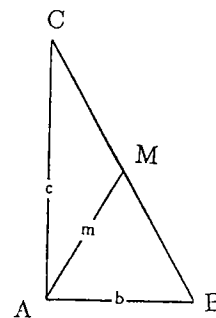


Figure 1

Now let us study the counterpart for a right tetrahedron. Let $ABCD$ be a right tetrahedron, where A is the vertex of the right angles (Fig. 2[a]). Let M be the centroid of the triangle BCD . Then one may define AM to be the median corresponding to the face BCD . Let AB , AC , AD , and AM be b , c , d , and m respectively. Then

$$m^2 = \frac{1}{9}(b^2 + c^2 + d^2). \quad (2)$$

(See "Tetrahedral Properties," page 2)

SEATTLE



'87

Mu Alpha Theta National Convention

Tetrahedral Properties

... FROM PAGE ONE

In order to prove (2), we observe that M is the point of intersection of the medians of the triangle BCD . Let L be the point of intersection of

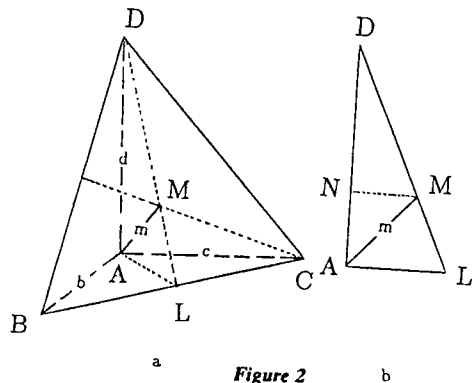


Figure 2

DM and BC . Then L is the midpoint of BC . Note that DAL is a right angle, and thus by (1) we have

$$AL^2 = \frac{1}{4}(b^2 + c^2). \tag{3}$$

Let us set the right triangle DAL face down (Fig. 2[b]). Note that ML is one third of DL . Draw MN perpendicular to AD . From the similar triangles DNM and DAL we obtain

$$\frac{NM}{AL} = \frac{DN}{DA} = 2/3.$$

This implies that

$$MN = \frac{2}{3}AL,$$

and from (3) we get

$$MN^2 = \frac{1}{9}(b^2 + c^2). \tag{4}$$

Now from (4) and

$$m^2 = AM^2 = MN^2 + AN^2$$

we get

$$m^2 = \frac{1}{9}(b^2 + c^2) + \frac{1}{9}d^2.$$

Consequently

$$m^2 = \frac{1}{9}(b^2 + c^2 + d^2).$$

Corollaries:

We observe that (1) implies that the locus of the vertex A of a right angle whose sides pass through two fixed points B and C is a circle whose center is M , the midpoint of the line segment BC , and its radius is given by (1) (Fig. 3). This circle passes through B and C .

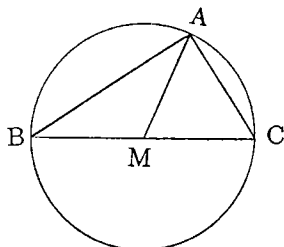


Figure 3

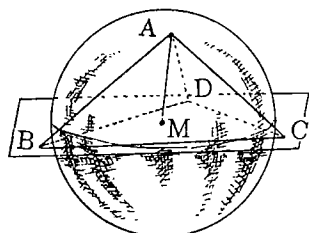


Figure 4

Similarly (2) implies that the locus of A the vertex of a right tetrahedron

whose edges of the right angles pass through three fixed points B, C , and D is a sphere whose center is M , the centroid of the triangle BCD , and its radius is given by (2) (Fig. 4). One can easily show that the points B, C , and D are outside of this sphere. For example, we shall show that $MD > AM$. Looking at Fig. 2[b], observe that

$$MD = \frac{2}{3}DL,$$

and

$$\begin{aligned} MD^2 &= \frac{4}{9}DL^2 = \frac{4}{9}(d^2 + AL^2) = \\ &= \frac{4}{9}\left[d^2 + \frac{b^2 + c^2}{4}\right] = \frac{1}{9}(4d^2 + b^2 + c^2). \end{aligned}$$

On the other hand

$$AM^2 = \frac{1}{9}(b^2 + c^2 + d^2)$$

which implies $MD > AM$.

2 The A Altitude

Let ABC be a right triangle, where A is the vertex of the right angle (Fig. 5). Let AH be the altitude corresponding to the hypotenuse. We shall

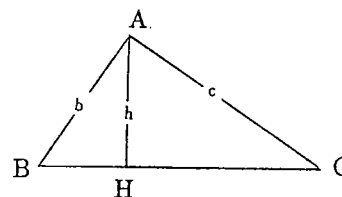


Figure 5

choose $AB = b, AC = c$, and $AH = h$. Then we observe that the area A of the triangle can be written in two ways

$$A = \frac{bc}{2} = \frac{ah}{2}, \tag{5}$$

where a is the hypotenuse. From (5) we obtain $h = \frac{bc}{a}$ which implies

$$h^2 = \frac{b^2c^2}{a^2} = \frac{b^2c^2}{b^2 + c^2}.$$

This equality is equivalent to

$$\frac{1}{h^2} = \frac{1}{b^2} + \frac{1}{c^2}. \tag{6}$$

Now we shall look into a similar equality for a right tetrahedron. Let $ABCD$ be a right tetrahedron, where A is the vertex of all the right angles

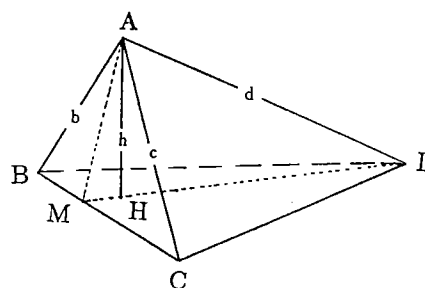


Figure 6

(Fig. 6). Let AH be the altitude corresponding to the face BCD , and let $AB = b, AC = c, AD = d$, and $AH = h$. Then

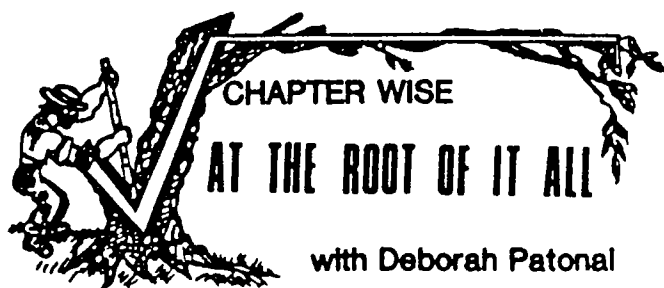
$$\frac{1}{h^2} = \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}. \tag{7}$$

The proof is quite straightforward. We draw the line DH . This line intersects BC at M , where AM and DM are perpendicular to BC (three perpendicular theorem). The triangle AMD is a right triangle, i.e., MAD is a right angle, and AH is the altitude of it corresponding to the hypotenuse

(See "Tetrahedral Properties," page 7)



WINNERS ALL--I! Paul A. Foerster, right, first winner of Mu Alpha Theta's Harold V. Huneke Distinguished Sponsor Award, is shown with Dr. Huneke at Miami National Convention. The popular sponsor from Alamo Heights High School, San Antonio, TX, was attending his 15th national meeting.



Mathematical Log Activities Editor Deborah S. Patonai is Mu Alpha Theta sponsor at Saint Vincent-Saint Mary High School in Akron--and a familiar figure at Mu Alpha Theta national conventions. Miss Patonai welcomes chapter news and views from sponsors and student members. Such submissions are intended to be the basis of this regular feature. Miss Patonai writes:

In this issue, *At the Root of It All* takes great pleasure in spotlighting two remarkable individuals, Paul Foerster and Anita Scott. Miss Scott, a 1986 graduate of Central High School in Tuscaloosa, AL, is this year's winner of Mu Alpha Theta's prestigious Robert Kalin Award. Mr. Foerster, a Mu Alpha Theta sponsor from Alamo Heights, San Antonio, TX, is the first recipient of the association's Harold V. Huneke Distinguished Sponsor Award. Both student and sponsor have devoted significant service and time to betterment of Mu Alpha Theta. Let us take a closer look at the accomplishments and lives of these two special people.

Always enjoying mathematics and problem solving, Anita received her first taste of formal mathematics competition when she joined her junior high school's math team. Later, as a sophomore, Anita became a member of Mu Alpha Theta, hoping to improve her math skills and to broaden her creativity in problem solving. Not only did her math techniques and scores improve, so did her opportunities. Through tournaments, competitions, and conventions at the local, state, and national level, Anita was given the chance to meet all types of Mu Alpha Theta people. She discovered that "most Mu Alpha Theta members are not merely exceptional math students but enthusiastic, well-rounded individuals as well." Anita delights in "interacting with people of similar interests, people who find pleasure in learning, knowing, and being challenged." In fact, these last words aptly describe Anita's character. She has grown and matured from the excitement and challenge of mathematics and competition provided by Mu Alpha Theta.

Realizing the vast rewards and benefits that Mu Alpha Theta offers, Anita wanted not only to participate in the

organization but also to lead it. This desire to lead such "an enthusiastic and talented group of people" resulted in Anita's being elected her chapter's president. Through her role as president, Anita has demonstrated great leadership and service. She has tried "to remind both members and non-members that math is fun by challenging them with puzzles-of-the-week." She has established standards for attendance and participation, sponsored a tutoring program, initiated a formal induction ceremony, and introduced other service activities. Along with her goal of making her club very active, Anita has "tried to promote interest and proficiency in mathematics in a school where, unfortunately, many students dread math classes."

As the first recipient of the Harold Huneke Award, Paul Foerster exemplifies all the qualities necessary for being an outstanding Mu Alpha Theta sponsor. Adjectives such as involved, caring, thoughtful, brilliant, devoted, demanding, and respected, describe this tall, friendly man, a familiar figure at 15 Mu Alpha Theta national conventions to date (all except New Orleans). In fact, my personal association with Paul goes back to my first national convention, 1977 in Dubuque, IA. While trying to keep out of the rain on board a Mississippi riverboat, Paul and Harold Huneke took time to talk to this young, naive, and inexperienced sponsor from Ohio. I was so impressed with these two gentlemen that I fell in love with Mu Alpha Theta.

Paul's connection with Mu Alpha Theta began in the mid 60's when the late Howard Wildman established a Mu Alpha Theta chapter at Alamo Heights. As a co-sponsor and later as the school's sponsor, Paul's involvement with Mu Alpha Theta increased. Envisioning "a summer meeting in which great students met with great mathematicians and let their minds interact," Paul, along with Wildman and Frank Allen, Mu Alpha Theta national president at the time, organized the first national Mu Alpha Theta convention. In San Antonio in 1968, a Mu Alpha Theta tradition was born! Paul served as the program chairman and later served temporarily as general chairman. From 1970 to 1973 Paul represented National Council of Teachers of Mathematics on Mu Alpha Theta's governing board.

With Paul as sponsor, Alamo Heights chapter is an active and productive math club. They hold meetings twice a month, one after school and one in the evening so that members in athletics and members who work can attend at least one meeting. Yearly they sponsor an invitational math tournament, where schools from all over Texas come to compete. This event is special since all questions are written by the chapter members themselves--with heavy editing, of course, from the sponsors. In addition to this

(See "At the Root of It All," page 8)



WINNERS ALL--II! Anita Scott, left, 1986 graduate of Central High School, Tuscaloosa, AL, and current winner of Mu Alpha Theta's Robert Kalin Award, is shown on stage at the Miami Convention being congratulated by national president Dr. Betty K. Lichtenberg, University of South Florida.

dia Log ue



with Log Editor Don Allen

Your Log Editor's Miami Convention "table hopping" has garnered a worthy duo of "favorite problems" to share with interested readers.

The first, number theoretic and concerned with divisibility and multiples, was provided by longterm Mu Alpha Theta supporter Adolph Holbrook, who teaches at South Pike High School, Magnolia, MS.

"A man's age is a multiple of his granddaughter's age for six consecutive years. How old is each in the sixth year?"

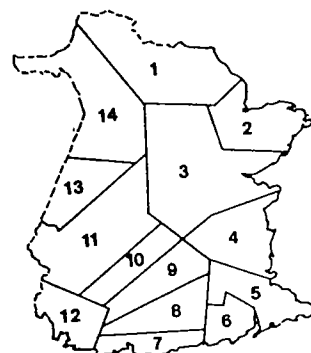
The second, a Diophantine problem leading to two relations in three "unknowns," but with implicit restriction to solution in nonnegative integers, was shared by Jackie Rosner, student member at North Miami Beach Senior High School. Figures are so delightfully outlandish that neither women's rightists nor child labor activists are likely to take the matter too seriously, so our concern is more directed to what Jackie meant, but didn't write, by jotting down ".05¢" ...

"In a certain factory, the workers are either men, women, or children. The men earn \$5 each, the women earn \$1 each, and the children earn .05¢ each. If there are a total of 100 workers, and the total payroll is \$100, then exactly how many are men, women, and children?"

Solution is instructive. Taking numbers of men, women, and children to be m , w , and c , respectively, and interpreting .05¢ as five cents, equations in numbers of workers and numbers of dollars are:

$$m + w + c = 100, \quad 5m + w + c/20 = 100;$$

subtraction reveals that 80 times the number of men must equal 19 times the number of children, with these numbers, necessarily, integers not exceeding 100. There are formal methods for solving systems of linear Diophantine equations, of course, but simple juggling sufficed to lead Jackie to the unique solution: 19 men, 1 woman, and 80 children. Do look up Diophantus in an introductory history of mathematics, and Diophantine equations as a chapter in Theory of Numbers.



Your Editor recently was privileged to share math club insights at annual teachers' meetings of Canada's "picture province," New Brunswick. Flying into Fredericton, the historic provincial capital, had us thinking maps and

(See "diaLogue," page 8)

BOOKS WRITTEN ESPECIALLY FOR FOR THOSE WHO TEACH, STUDY, AND ENJOY MATH

The Pythagorean Proposition, by *Elisha S. Loomis*. A historical review, presenting 370 demonstrations of the Pythagorean theorem. Vol. 1 in the NCTM Classics series. Hardback. 306 pp.; #155N8; \$12.00.

Polyhedron Models for the Classroom, by *Magnus J. Wenninger*. Second edition has seven removable pages of designs to be used as templates to make the models shown in the text; directions and historical notes. 80 pp.; #149N8; \$4.00.

Some "Prime" Comparisons, by *Stephen I. Brown*. A mind-expanding journey into number theory for intellectually alive high school students or students in teacher education courses. 106 pp.; #265N8; \$6.00.

Whom the Gods Love: The Story of Evariste Galois, by *Leopold Infeld*. Fascinating biography of a brilliant, passionate young nineteenth-century mathematician; first published in 1948. Vol. 7 in the Classics series. Hardback. 323 pp.; #248N8; \$13.00.

Topics for Mathematics Clubs, 2d ed., edited by *LeRoy C. Dalton and Henry D. Snyder*. Stimulates interest in mathematical investigation through exciting topics not usually discussed in the classroom; organized to give a feeling for each topic as a whole through student presentation of related subtopics. Bibliographies provide for further reading. 106 pp.; #169N8; \$6.00.

Mathematics through Paper Folding, by *Alton T. Olson*. Offers active experiences in discovering and demonstrating mathematical relationships. 64 pp.; #60N8; \$4.00.

Mathematical Challenges II—plus Six, edited by *Thomas Hill*. Includes 100 problems selected from the *Mathematics Student Journal* since 1965; plus six entertaining articles, three by high school students. 128 pp.; #223N8; \$6.00.

The Trisection Problem, by *Robert C. Yates*. A treatise on one of the three famous problems of antiquity. Bibliography. Vol. 3 in the Classics series. Hardback. 68 pp.; #159N8; \$6.75.

How to Develop Problem Solving Using a Calculator, by *Janet Morris*. Perforated activity pages organized under the topics of whole numbers, decimal numbers, percents, and geometry; can be used with upper elementary grades through high school. Dedicated to the proposition that calculators free children to think out problem solutions. 40 pp.; #112N8; \$4.00.

Four-Dimensional Geometry—an Introduction, by *Adrien L. Hess*. Includes history and definition of 4-D geometry, selected drawings and models, suggestions on how to study the configurations, and applications for high school students. 32 pp.; #88N8; \$3.00.

Geometry Problems My Students Have Written, by *Ruth C. Kespohl*. Clever illustrations clarify problems and solutions written by high school students. 87 pp.; #247N8; \$6.00.

Historical Topics in Algebra. Contains 25 capsule discussions of the interesting personalities, situations, development, and origins of topics in algebra. Gives life and meaning to the topics of today with relevant information from the past. The capsule format puts appropriate material at your fingertips for immediate classroom use. 74 pp.; #129N8; \$5.00.

Mathematics and Humor, edited by *Aggie Azzolino, Linda Silvey, and Barnabas Hughes*. A collection of jokes, riddles, and cartoons to add levity to bulletin boards and test papers and pique the interest of students from junior high school up. 58 pp.; #266N8; \$4.00.

Mathematics and Science: An Adventure in Postage Stamps, by *William L. Schaaf*. The influence of mathematics on technology and thus on civilization as seen through postage stamps. 152 pp.; #100N8; \$9.00.

Mathematics Projects Handbook, (2d ed.), by *Adrien L. Hess*. Useful guide for junior and senior high school teachers and students in choosing and developing projects; extensive bibliography. 48 pp.; #91N8; \$3.75.

Dancing Curves: A Dynamic Demonstration of Geometric Principles, by *Merwin J. Lyng*. Instructions for constructing a string model to be used with light beams to dynamically illustrate conic sections as well as lines, curves, and surfaces. Includes four color slides. 16 pp.; #240N8; \$4.05.

Deductive Systems: Finite and Non-Euclidean Geometries, by *Garth E. Runion and James R. Lockwood*. Two non-Euclidean systems are considered as a means of strengthening the student's awareness and understanding of the deductive systems of mathematics in general. For high school and college levels. 90 pp.; #243N8; \$5.00.

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Chapter Challenge

'KEY PHRASE' CIPHER OCTET
RICH IN MATH THOUGHT

By Don Allen

The mind that cracks a cipher for sheer pleasure not infrequently is the mind that can find joy equally in language and in mathematical-logical thought. Accordingly, mathematics clubs, as we know them, tend to like being introduced to--and beating!--a new cipher. As an instance, the exotic "ragbaby" cipher, presented to Mathematical Log readers through cryptographic reflections on Mu Alpha Theta's outstanding Hawaii Convention (MLog 29:4, December 1985), proved popular with chapters as far afield as--fittingly--Honolulu's Kamehameha Schools (Tall Timbers #13, December 1985). "Ragbaby" strategies remain accessible (as Log copies or on microfiche) as a solid challenge to the chapter that thrives on such group efforts. The cipher being shared this issue is the "key phrase," quite different from forms likely to be familiar to Log readers but easily learned and a recreational favorite in cryptographic circles for some decades.

Gaines, in her Cryptanalysis (Dover, 1956, unabridged from 1939 ed.), treats "the celebrated key-phrase cipher" as an already (1939) familiar and established cryptographic form (p. 103).

The American Cryptogram Association, in its "AGA and You" supplement for new members (The Cryptogram, XL, 3, May-June 1974, part 2), provides technical specifications for key phrase constructions, recommending a message of 75-100 words.

The Mathematical Log will describe and demonstrate the key phrase as a cipher form, share insights into construction and solution, then leave the reader with a set of new "mathematical" encipherments calculated to reward the solver for the time and effort necessary for the task.

A Mu Alpha Theta "best buy" for your chapter, school or college, or community library: The Mathematical Log, 29 years complete, all for US\$10, from National Office!

The key phrase essentially is a "substitution cipher," replacing letters in the plaintext (pt) by corresponding ciphertext (ct) symbols--but key phrase construction technique is distinctive and the resulting letter-to-letter correspondence will not be one-to-one. Key phrase encipherment begins with some distinctive, easily-remembered 26-letter "phrase"--for example,

OCTAGONAL BASED PYRAMIDS RARE.

The phrase is written out, with (letter by letter) the normal alphabet above it.

(pt) abcdefghi jklmn opqrstuv wxyz
(ct) OCTAGONAL BASED PYRAMIDS RARE

Note that, in this correspondence, the cipher A could represent d, h, k, r, or x; the G represents e; and H has no plaintext equivalent. Letters do not normally "stand for" themselves in this sort of recreational cryptography. Key phrases are so selected that self-representation does not occur.

Enciphering, using the above key phrase and cipher alphabet, presents no difficulty. Icosahedron becomes
icosahedron
LTPMOAGAAPD.

Deciphering could seem to present enormous problems, but in practice does not. Thus, arguing combinatorically from multiple representations, ARYGACPSO, a mathematical term, could give rise to $5 \times 3 \times 1 \times 1 \times 5 \times 1 \times 1 \times 2 \times 2$, or 300 possibilities.

In practice, a quick scan of

ARYGACPSO
d d
hq h la
kwpekbo_{vf}
ry r
x x

renders "hyperbola" apparent.

The enciphering technique is quickly mastered. Use the key phrase of the proper length, KEY PHRASE OF THE PROPER LENGTH to encipher the plaintext:

The key phrase is a clever substitution cipher, easy to employ but challenging in its distinctive provision for multiple correspondences.

You should obtain:

RSH FHT RSPKEH EE K YTHEP ELEERERLREPE YERSHP, HKET
RP HHRTP ELR YSKTTHEAEEA EE ERE PEEREYREEH RPPEEEPE
RPP HLTRETH YPPHERPEPHEYHE.

Key phrase decipherment, conversely, calls for a honing of self-developed solving skills--"discovery learning" in the very best sense! Practice, if you will, on this key phrase "demonstrator": we've entitled it "Good variety." Your "tip"--overly generous, very possibly--is that the word "achievement" occurs in the plaintext, necessarily as the twentieth (last) word, the only eleven-letter word. Cipher equivalents of nine different plaintext letters, and as many letters of the vital key phrase, in proper positions, are given by this one-word clue.

Good variety. (Tip: achievement) APEX
K RCOO OKFAH UR YOENEUAOKNSEY "EENHD" YKODD RUO IKOEHP
DHE SUPD UR KEEKYO ECE DHOIHD EU EUODEHO DUOIHOD DHPFDH
UR KYSEIHDFE.

A hyphen in ciphertext divides a word, but not necessarily between syllables. A hyphen in plaintext is shown as an equal sign (=) in ciphertext.

An asterisk preceding a word in ciphertext indicates a capitalized proper noun or proper adjective.

Word 12, above, provides a useful opening, for we know possibilities for each of the first five letters: KEEKYO = attac-, suggesting O = k. Word 10, too, is potentially revealing, DHE SUPD beginning meih-- or meth--. Our key phrase, even without these added insights, runs

K-Y-H--SE---DF-----E-I-----,

and its first nine letters might suggest a radical, but in this instance right, guess.

Working alone or--better, we think--with congenial companions, sustained effort and reflection can lead to rather remarkable cipher-solving skills. Develop such skills, if you will, on the following key phrase octet, constructed for Mathematical Log inclusion and featuring a decided mathematical flavor.

Our Mathematical Key Phrase Octet

1. The impossible takes longer. (Tip: circle) APEX
MCAOGIMAET UETHG, "OEPGCAET" IACIHG, "NPGHAIUMAET" IP-SG, SUOIAEUMGN *TGGGG TGDTCMGCO, SRD SUAHGN MD TCUOG ATGDOOASAHAME DS MUOGO.
2. Euclid never suspected! (Tip: result) APEX
ARLSAUSUIR *LTAIRN MERTARL ITA MAESASIR SASIR MAELRRM-TAL TSL MEAA=TI=RRAMEAN SRL NERIPEAS URSEMEIEI, REEMR EARIERMRP ARLEIM.
3. Textbook example. (Tip: range) APEX
EOEAEI "FONNOV" NFANRNE AE ENEVEFR VWOFVNE RNO FNIAE-FENER *RNAEFOA VA INDENEAN AOVW EOIEOEOVOET NFANNFV-ONI, AONN FOETN AE ONNROEVOEAEI.

(See "Cipher Octet," page 8)

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Editorially Logged
For Champions

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Ever wonder what the test questions look like in a world-class mathematical olympiad? Wonder if you'd understand the meaning of the questions, or make headway toward one or more solutions? Here is your chance to find out! An exchange copy of the Newsletter of the World Federation of National Mathematics Competitions, published in Australia, has reached The Mathematical Log, and in it are the English-language versions of the six questions which constituted the 27th International Mathematical Olympiad, as held in Warsaw, Poland, 9-10 July 1986.

Questions 1-3 were written on the first day, with the time allowed 4.5 hours. Questions 4-6 were written on the second day, also in 4.5 hours. Questions were of equal value. Solvers from 37 nations found Question 3 the most difficult, followed by Questions 6, 4, 1, 2, and 5.

Exceptionally interesting mathematical reading, the six Olympiad questions were as follows:

1. Let d be any positive integer not equal to 2, 5, or 13. Show that one can find distinct a, b in the set $\{2, 5, 13, d\}$ such that $ab - 1$ is not a perfect square.

2. A triangle $A_1A_2A_3$ and a point P_0 are given in the plane. We define $A_s = A_{s-3}$ for all $s \geq 4$. We construct a sequence of points P_1, P_2, P_3, \dots such that P_{k+1} is the image of P_k under rotation with center A_{k+1} through angle 120° clockwise (for $k = 0, 1, 2, \dots$). Prove that if $P_{1986} = P_0$ then the triangle $A_1A_2A_3$ is equilateral.

3. To each vertex of a regular pentagon an integer is assigned in such a way that the sum of all the five numbers is positive. If three consecutive vertices are assigned the numbers x, y, z respectively and $y < 0$ then the following operation is allowed: the numbers x, y, z are replaced by $x + y, -y, z + y$ respectively. Such an operation is performed repeatedly as long as at least one of the five numbers is negative. Determine whether this procedure necessarily comes to an end after a finite number of steps.

4. Let A, B be adjacent vertices of a regular n -gon ($n \geq 5$) in the plane having center at O . A triangle XYZ , which is congruent to and initially coincides with OAB , moves in the plane in such a way that Y and Z each trace out the whole boundary of the polygon, X remaining inside the polygon. Find the locus of X .

5. Find all functions f , defined on the non-negative real numbers and taking non-negative real values, such that:

- (i) $f[x f(y)] f(y) = f(x + y)$ for all $x, y \geq 0$,
- (ii) $f(2) = 0$,
- (iii) $f(x) \neq 0$ for $0 \leq x < 2$.

6. One is given a finite set of points in the plane, each point having integer coordinates. Is it always possible to color some of the points in the set red and the remaining points blue in such a way that for any straight line L parallel to either one of the coordinate axes the difference (in absolute value) between the numbers of blue points and red points on L is not greater than 1? Justify your answer.

Such, we assert, are the math questions of champions! Superb Mu Alpha Theta "food for thought." H.D.A.

abcabc?

More our speed, we confess, is Charles W. Trigg's challenge in the current (January) School Science and Mathematics (Problem 4124), "In the system of enumeration having base eleven, find a six-digit square of the form abcabc."

A pleasant evening of doodling we found that to provide!



SEATTLE '87
Mu Alpha Theta National Convention

Mu Alpha Theta's 17th National Convention is slated for 1st-5th August at University of Washington, Seattle. For details contact Paul See, address above or (206) 842-2634.

