PLAN NOW FOR MIAMI – AUGUST 3-7

The Mathematical Log

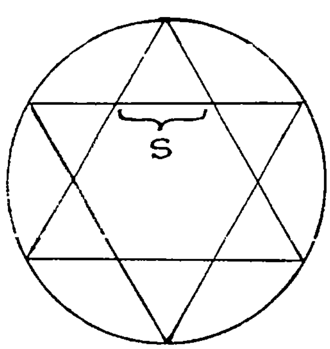
Volume 30, Number 1 February 1986

HIGH SCHOOL CONTEST

Continuing Variety Feature Of Competition Problems

Mathematical problem solving, in its finest sense, represents one of the better ways to promote growth and maturity in mathematics and to consolidate and reaffirm growth already achieved. On this we're agreed! Further, problem solving has been and remains a favorite Mu Alpha Theta convention, chapter meeting, and individual activity--and a continuing store of good problems can be a challenge to find. Accordingly, The Mathematical Log takes pleasure in sharing herewith competitive questions from the current (28th annual) University of Santa Clara High School Mathematics Contest, as graciously provided by Santa Clara's Gerald L. Alexanderson. Clearly presented solutions and contributions to solutions would receive credit in the competition, with all work to be shown.

1. A six-pointed regular star is inscribed in a circle of radius a. Find the length s of the side of the regular hexagon formed inside the star, in terms of a.



2. (a) What is the largest integer that always divides (evenly)

$$n(n^2 - 1)(n^2 - 4)$$

for every integer n, $n \geq 2$.

(b) What is the largest integer that always divides (evenly)

$$n(n^2 - 1)(n^2 + 1)$$

for every integer n, $n \geq 2$.

3. Write a simple expression in k for the following:

$$\log_2\left(\log_2 \underbrace{\sqrt{\sqrt{\sqrt{\cdots\sqrt{2}}}}}_{k\ \text{times}}\right)$$

4. Factor 8051 into two prime factors.

5. In a three year high school 4/11 of the students in the school are sophomores, several sevenths are juniors and the remaining 324 students are seniors. If there are s students enrolled in the school, find s.

6. A town is in the form of a square grid. Streets running north and south are numbered 1, 2, 3, ... and the east-west streets are lettered A, B, C, ... To go from home, which is at 8th and P streets, to school, at 15th and V streets, one has normally a choice of 1716 different routes to choose from, if at each intersection one always moves to a street with a higher number or a later letter of the alphabet.

(a) One morning we discover that the block between 12th and S and 12th and T is closed for repairs. How many of the 1716 routes are no longer possible?

(b) If the block of part (a) and the two blocks between 10th and S are all closed, how many of the original 1716 routes are no longer possible?

NOMINATIONS INVITED FOR NATIONAL AWARD

--SEE PAGE 2.

7. (a) Find the number of ways three distinct numbers can be chosen from the set $\{1,2,3,4, \ldots, 3n\}$ such that the sum of the three numbers is a multiple of 3.

(b) Generalize to choosing sets of five from $\{1,2,3,4, \ldots, 5n\}$ where the elements of the subset add to a multiple of 5.

Readers with an interest in contest questions are reminded that the first twenty-five years of this competition are available as The Santa Clara Silver Anniversary Contest Book (272 pages, $18.50) from Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303.

TO HONOR HAROLD HUNEKE

Nominations Invited For Student Award

The Mu Alpha Theta Board of Governors has announced a new student award, to be presented for the first time at National Convention in Miami.

To honor Harold V. Huneke an award in his name will be presented to the student who best follows his example of loyalty and dedicated service to Mu Alpha Theta.

According to Dr. Thomas J. Hill, Huneke's successor as national Secretary-Treasurer, governors will be seeking "the member who is the worker, the quiet one who often is in the background getting the job done while someone else may be getting the attention."

Winners of the award are to be selected by national student officers (this year's were chosen at Honolulu convention) and by the Governing Council. The first Harold Huneke Award is to be presented at the opening session of national meetings in Miami.

Each Mu Alpha Theta chapter will receive a nomination form, a letter describing the award, and the procedure for selecting the nominee.

"Be sure your chapter is represented," national officers urge.

LEWIS CARROLL TRADITION

CIRCLE - SQUARING STRATEGY DEVISED BY SPONSOR

"Squaring a circle?" Impossible, of course, should one be limited (arbitrarily, but not unusually) to the rules and the tools of classical Greek geometry.

Like "trisecting an angle" or "duplicating a cube," "circle squaring" is demonstrably impossible by straightedge and compasses procedures (those sanctioned by Euclid)--but no problem, in practice, once the right tools are devised and put to use.

Correspondingly, recalling Lewis Carroll's "Doublets" activity (mathematician Lewis Carroll of Alice fame, see December Log, p. 6), transforming CIRCLE into SQUARE by a finite sequence of "linking words," each word differing from the preceding by exactly one letter, is blatantly impossible--unless one creatively stretches a rule and devises a new and clever variant of Carroll's Victorian wordgame.

Mu Alpha Theta sponsor Gary D. Shaffer at Allegany Community College (Willow Brook Rd., Cumberland, MD 21502) did just that when he conceived and implemented the following remarkable line of attack:

C I R C L E
E C L A I R
C L E A R S
S C R E A M
S T R E A M
S M A R T S
S T A R T S
S T U A R T
Q U A R T S
S Q U A R E

The variation on Lewis Carroll's "Doublets" approach is that Gary Shaffer changed one letter, then permitted permutation (rearrangement) of letters, at each stage's linking word--which certainly does the trick.

Shaffer defends STUART, the seventh linking word in the CIRCLE-SQUARE transformation, as "of or relating to the Scottish royal house to which belong the rulers of Scotland from 1371 to 1603" [Webster's New Collegiate Dictionary, 1977]--reasonable, unless proper nouns and adjectives were to be entirely ruled out.

Now, HYPERBOLA to TESSERACT ... which adds a new dimension (or two!) to a significant variation on (we'd say improvement on) the math-word aspect of the traditional Lewis Carroll challenge. --H.D.A.

CHALLENGE TO CHAPTERS--and "something different"--"Cipherscrambles," a unique combination of rapid-fire arithmetic (look for shortcuts--valid shortcuts!) and math-related anagramming (rearranging letters to give familar math-words--against the clock) ... featured in Mathematical Tall Timbers #14, going out to chapters this month.

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STRATEGY NEEDED

MAKING SQUARES 'MAGIC' SEEN WORTHY PROJECT

By Ali R. Amir-Moéz

Every so often one should delve into those "tried and true" topics known to be rich in mathematical insights--in part to make them accessible to a new generation. One such topic is "magic squares," a very old subject and one on which an enormous literature exists. This expository note will present some easy techniques for constructing such number squares. Proofs are being omitted, to keep length within bounds, but references are appended, to encourage further investigation.

Most students in Grade VII or VIII may, by trial and error, have chanced upon the solution of the 3x3 magic square--a common "enrichment" item in upper elementary school math texts. Except for rotations and reflections, the solution (Fig. 1) is unique.

8	1	6
3	5	7
4	9	2

Fig. 1

One already knows what rows, columns, and diagonals are. The rows of a magic square are numbered from top to bottom, 1st, 2nd, etc., and the columns are numbered from left to right. We have no occasion for naming the diagonals, but the ones from corner to corner are "main diagonals."

1. Odd Squares. Odd squares are 3x3, 5x5, 7x7, etc. Let us study a method of construction due to De la Loubère. First we show it for a 3x3 square (Fig. 2). We write 1 in the middle cell of the 1st row, and we proceed diagonally upward, from left to right. Since 1 is on the 1st row, one proceeds to the bottom of the 3rd column, and writes 2. From this cell we go to the left of the 2nd, and write 3. We can go no further. We refer to this as getting into an obstacle. In Fig. 2, arrows indicate direction, and an obstacle is given by a dot and a circle. When we encounter an obstacle, we drop directly below that cell, then enter the next integer (Fig. 3). Then we continue. Again, at the upper right hand corner, we have an obstacle, and drop below it and write 7. Then we continue as before (Fig. 4). Thus we complete a 3x3 magic square. The sum ("magic constant") for each row, each column, and each diagonal, is 15.

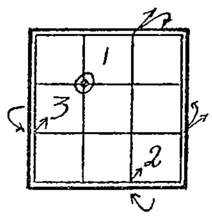

Fig. 2

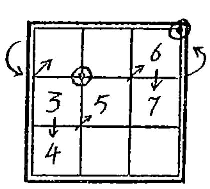

Fig. 3

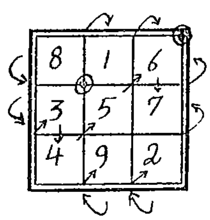

Fig. 4

Now we replicate this construction technique for the 5x5 magic square. Since the meaning of arrows and obstacles is already understood, a diagram (Fig. 5) will suffice. The reader may calculate the magic constant, then test sums for rows, columns, and diagonals. Knowing De la Loubère's technique, the reader may go on to other odd magic squares.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Fig. 6

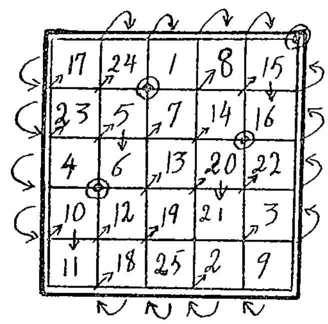

Fig. 5

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Fig. 7

2. Doubly-Even Squares. Doubly-even squares are 4x4, 8x8, 12x12, etc. We shall first study a 4x4 magic square (Fig. 6). We write 1 through 16 in their natural order. We then reverse the integers in the main diagonals (Fig. 7.) This, actually, means that elements of diagonals are substituted for--replaced--by their complements. Let us define the complement of each integer. We write the integers 1 through 16, as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16. The two numbers which are the same distance from the end are said to be complements. For example, (1, 16), (2, 15), (3, 14) are pairs of complements. In general, for any set of integers 1, 2, ..., n , we may say (k, n - k + 1), k = 1, ..., n, is a pair of complements. This magic square (Fig. 7) has more interesting properties than the usual row-sum, column-sum, and diagonal-sum. The sum of the numbers in the four corners is also 34. The sum of the numbers in the four middle squares is also 34. One can easily find other such properties. We shall leave to the reader a more thorough investigation of this magic square.

Mathematics Editor of The Mathematical Log--and longtime friend of Mu Alpha Theta--Dr. Amir-Moéz teaches at Texas Tech University. Interested readers may contact him there, c/o Department of Mathematics, P.O. Box 4319, Lubbock, TX 79409.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Fig. 8

64							57
	55					50	
		46			43		
			37	36			
			29	28			
		22			19		
	15					10	
8							1

Fig. 9

Now let us look to an 8x8 magic square. A similar

(Continued on page 4)

"MAGIC"

. . . FROM PAGE 3.

technique is employed. First we write 1 through 64 in their natural order (Fig. 8). Then we reverse the main diagonals (Fig. 9); i.e., we replace by its complement each element of the diagonals. Next we interchange the two minor diagonals indicated in Fig. 9 by dotted lines, and then reverse them (Fig. 10); i.e., elements of these diagonals are replaced by their complements. We do a sim-

64			61				57
	55	54				50	
	47	46			43		
40			37	36			
			29	28			25
		22			19	18	
	15				11	10	
8				4			1

Fig. 10

64	2	3	61	60	6	7	57
9	55	54	12	13	51	50	16
17	47	46	20	21	43	42	24
40	26	27	37	36	30	31	33
32	34	35	29	23	38	39	25
41	23	22	44	45	19	18	48
49	15	14	52	53	11	10	56
8	58	59	5	4	62	63	1

Fig. 11

ilar modification to the diagonals indicated in Fig. 10--obtaining an 8x8 magic square (Fig. 11). Note that what we have done amounts to interchanging elements of the diagonals and minor diagonals with their complements.

One can extend this technique to other doubly-even magic squares.

3. Singly-Even Squares. A magic square of 6x6, 10x10, 14x14, etc., is called singly-even; in other words it has $[2(2m + 1)] \times [2(2m + 1)]$ squares.

Let us study the construction of a 6x6 magic square. We divide the square into four 3x3 squares (Fig. 12). On the upper left corner we repeat the magic square of Fig. 1. Adding 9 to each element, we write the result in the 3x3 square in the lower right corner; i.e., the continuation of consecutive integers from 10 on. Then we continue the procedure for the upper right 3x3 square; i.e., we add 9 to every element in the lower right square and enter the result in the corresponding cell in the upper right. Finally, remaining integers through 36 go into the 3x3 lower left square, again adding 9 to the element in the corresponding cell. Note that the sum of the numbers in each column is 111, as expected, since

$$1 + 2 + \ldots + 36 = \frac{36(37)}{2} = 666.$$

8	1	6	26	19	24
3	5	7	21	23	25
4	9	2	22	27	20
35	28	33	17	10	15
30	32	34	12	14	16
31	36	29	13	18	11

Fig. 12

35	1	6	26	19	24
3	32	7	21	23	25
31	9	2	22	27	20
8	28	33	17	10	15
30	5	34	12	14	16
4	36	29	13	18	11

Fig. 13

Now, examining the sums for the rows, we observe that for the first three rows the sum for each row is 84; i.e., 27 short of 111. For the three lowest rows the sum is 138, 27 more than 111. To remedy this, we interchange 8, 5, 4, with 35, 32, 31, respectively, since

$$35 - 8 = 32 - 5 = 31 - 4 = 27.$$

Thus we obtain a 6x6 square for which the row sum and the column sum are the same, and each sum is 111 (Fig. 13). The reader may check the diagonals.

A similar technique can be used for constructing a 10x10 magic square or larger singly-even square. We include the result for a 10x10 square (Fig. 14). The reader may refer to Fig. 5 and discover which terms have been interchanged.

92	99	1	8	15	67	74	51	58	40
98	80	7	14	16	73	55	57	64	41
4	81	88	20	22	54	56	63	70	47
85	87	19	21	3	60	62	69	71	28
86	93	25	2	9	61	68	75	52	34
17	24	76	83	90	42	49	26	33	65
23	5	82	89	91	48	30	32	39	66
79	6	13	95	97	29	81	38	45	72
10	12	94	96	78	35	37	44	46	53
11	18	100	77	84	36	43	50	27	59

Fig. 14

Other Techniques. There are many ways for constructing magic squares. We conclude by presenting an interesting approach for odd squares.

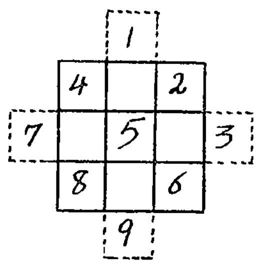

Fig. 15

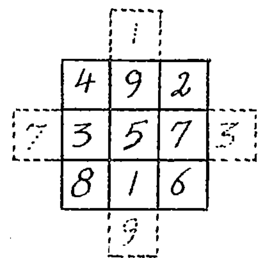

Fig. 16

Let us go back to the 3x3 square. We add a single square to the middle of each side (Fig. 15). Then we write 1 through 9 diagonally, as in Fig. 15. Next we carry those numbers which are outside the main square to the opposite sides (Fig. 16). The reader will recognize that this is a magic square.

We shall repeat the method for a 5x5 square. Here we add four squares to each side of the main square (Fig. 17).

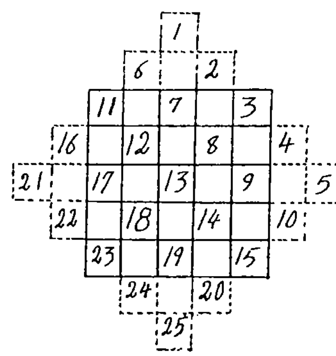

Fig. 17

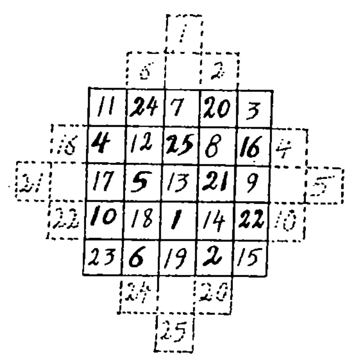

Fig. 18

We write 1 through 25 diagonally, as in Fig. 17. Then, as in the previous instance, we translate outside numbers to empty squares of the opposite side (Fig. 18). We obtain in this manner a 5x5 magic square. The reader may wish to extend the procedure to a 7x7 or larger "odd" square.

There are, as we have noted, many methods that will yield magic squares. The reader would do well to refer to some of the available references.

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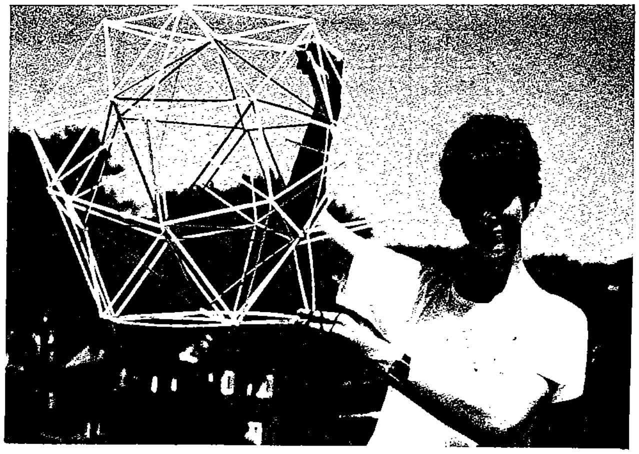
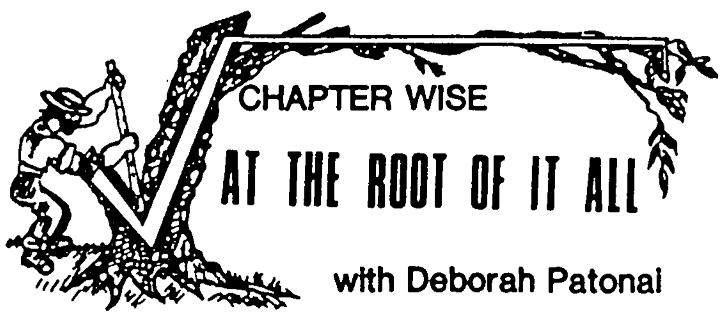


ART AND SCIENCE blend most strikingly in this stellated elaboration of a skeletal regular icosahedron. Constructed of drinking straws and thread by Nathaniel Fink, above, a 1985 Summer School student of the Editor at Canada's McGill University, it reflects a popular Chapter activity.



√At The Root Of It All, Debbie Patonai's regular compilation of news and views of the chapters and the people that are Mu Alpha Theta, deservedly has become one of our most popular Mathematical Log features. Miss Patonai is Mu Alpha Theta sponsor at Saint Vincent-Saint Mary High School in Akron (see Log masthead)--where she'd be delighted to receive news of your school and chapter.

For this February Mathematical Log, Debbie writes:

Mu Alpha Theta has a tendency to touch the lives of many individuals who work within its structure. Chapters, members, sponsors, and national and state officers of this great organization often find Mu Alpha Theta spinning and weaving its ways into their everyday existence without their being fully conscious of its effect. √At The Root Of It All tries to uncover people who have been brushed by this finger of Mu Alpha Theta. In this issue the column will spotlight a sponsor-governor from Illinois, a fund-raising effort in Florida, and a message from our Mu Alpha Theta national president.

First of all, congratulations are in order for Richard Rhoad, sponsor of the Mu Alpha Theta chapter at New Trier High School in Illinois and Governor for the Northeastern Region of Mu Alpha Theta. This Fall, Richard was chosen to receive the 1985 Presidential Award for Excellence in Mathematics Teaching for his State. This prestigious award is sponsored by the National Science Foundation to identify and suitably recognize excellence in mathematics and science teaching. Richard, with one mathematics and one science teacher from each state, received an expense-paid trip to Washington to receive the award, and a $5000 grant for his school. As the opening speaker at Mu Alpha Theta's 1985 National Convention in Honolulu and as a featured speaker at countless other such gettogethers, Richard has captivated many audiences of young people and adult educators--in Hawaii he beautifully blended mathematics and song! Through a personal friendship gained through Mu Alpha Theta, I know of Richard's love for his students and of his devotion to our organization. Richard indeed richly deserves such an award ... and we extend heartiest congratulations.

Recognition can, of course, take on many forms. Travelling now to Florida state, the column now focuses on how a Mu Alpha Theta chapter at Plant City High School earns $2500 indirectly ... from its school's annual Calendar Girl contest. Although the competition is not sponsored by the Mu Alpha Theta chapter, through efforts of the club sponsor, David Steele, the chapter reaps the benefits.

Each year Plant City High School stages a beauty pageant to select 13 girls to grace the pages of the School's calendar. David Steele, along with colleagues Don Wade and Jim Stewart, ran this past year's extravaganza in November. Any girl with at least a 2.0 GPA could apply and enter the first round of competition--where 21 semifinalists are picked. These 21 advance to a further round where the 13 Calendar Girls are chosen: one is named as the Cover Girl, each of the others represents a month of the year. Income from the school's Calendar Girl Contest includes application fees, admission tickets for the event, and Calendar advertising. After expenses, Mu Alpha Theta nets about $2500, the Mathematics Department Computer Fund $750, and the School's stage and singing groups, the rest. Even though this year was David Steele's first experience of running the show, he said it was easier than selling the 9000 candy bars that his busy Chapter sold last year!

Our final comments this issue come from Mu Alpha Theta's genial president, Betty Lichtenberg. Betty Lichtenberg is Professor of Mathematics Education at University of South Florida in Tampa. She has been active in Mu Alpha Theta for many years, and has served as Editor of The Mathematical Log. She firmly believes that Mu Alpha Theta is the best! At the risk of sounding like a mathematics cheerleader, she explains: "This honor society in school mathematics draws from the most academically talented students in the country. Its members exemplify the leadership qualities of congeniality and willingness to work." Only 20- to 30 000 students currently have the privilege of being in this organization. Betty Lichtenberg challenges us to "spread the word" of Mu Alpha Theta. What a lasting and valuable impact we could have were we significantly to increase the numbers of chapters and members.

Illustrating those thoughts of Mu Alpha Theta's national leader, Mu Alpha Theta sponsors Richard Rhoad and David Steele have been "touched" by Mu Alpha Theta!

At a Loss for Words? Relax, Do a Problem

"Ragbaby" ciphers (so called) being more "fun" than "useful" in intent, Logmaster Todd Belton's chance discovery of an ingenious "usefulness" comes as an agreeable surprise. Ragbabies, Todd reports, are an antidote to "writer's block."

The Baton Rouge, LA, Mu Alpha Theta enthusiast spent December holidays "working on a 100-page story I had sworn to finish," but the last 40 pages were "proving troublesome." Hence the ragbabies.

"When afflicted with writer's block, I turn my attention to something else for a while," Todd tells us. "'Ragbabies' are indeed 'fun' puzzles ... they also proved very handy in this connection.

Todd submits his solution to Hawaii-related Ragbaby #2 (December Log), "Time travel." Keywords: LONGEST DAY.

TORRID ZONE HIGH SUN WARMING FLESH AND BONES ACHING FOR OVERDUE NIGHT REPOSE ATTEST TO TIME DIFFERENCES, TROPIC DELINEATIONS AS ALL TOO REAL, NOT MERE CARTOGRAPHIC CONVENTIONS.

Todd's correct solution called for "fifteen minutes of prolonged (agonized) thought."

"Just because I finished it quickly doesn't mean it was easy," Todd writes, "... but it was fun, and I intend to work on the other eleven when I've the chance. The story, unfortunately, has to come first!"