The Mathematical Log

MU ALPHA THETA

VOLUME 30, NUMBER 4 DECEMBER 1986

Andree Award Created

Enhanced Award Structure Will Recognize Students, Sponsors, Prospective Teachers

A three-fold system of recognition and support of outstanding people within Mu Alpha Theta and of prospective teachers of school mathematics was approved by the Mu Alpha Theta governing council at Miami Convention meetings. Provided for, in addition to the Robert Kalin student award, were the Harold V. Huneke award, redefined as a Distinguished Sponsor Award, and the Richard V. Andree Award for the Pursuit of Mathematics Education.

The three awards bear the names of leaders with outstanding foresight within Mu Alpha Theta and the mathematics education community, and a growing list of recipients should underline individual excellence in three areas of the organization's strength.

The award to a member or former member who proposes to embark on a career in mathematics teaching was first suggested at Miami Convention and immediately acted upon by Mu Alpha Theta governing council. Recent convention speakers have reiterated the growing shortage of highly qualified mathematics teachers, and the challenge and career potential of the area. Council voted to name the new award in honor of Richard V. Andree, an exemplary mathematics educator who, with Josephine Andree, founded the Mu Alpha Theta movement in the 1950s. The first Andree Award will be made at Mu Alpha Theta's 1987 Seattle convention.

The Harold V. Huneke Distinguished Sponsor Award was conferred for the first time at Miami convention. Recipient was Paul A. Foerster, convention "regular" and long-time sponsor at Alamo Heights High School, San Antonio, TX. Foerster received the award from Harold Huneke, former Mu Alpha Theta secretary-treasurer and ambassador-at-large, on recommendation of the governing council.

Mu Alpha Theta's annual outstanding student award, conferred upon a member in attendance at national convention, was provided for by mathematics educator Robert Kalin at the conclusion of his Mu Alpha Theta presidency, 1977-79. Dr. Kalin at that time expressed the desire to encourage and support a worthy student member with a contribution from textbook royalties. Governing council drew up the now-familiar nomination and selection procedures, and voted that the award be known as the Kalin Award.

The Kalin Award winner at Miami convention was Anita Scott, a 1986 graduate of Central High School, Tuscaloosa, AL.

A personal introduction to Huneke Award winner Paul A. Foerster and Kalin Award winner Anita Scott will be the content of Activities Editor Deborah S. Patonia's popular column in February's Mathematical Log. Detailed particulars on Kalin, Huneke, and Andree nomination and selection procedures will be sent to all sponsors by Mu Alpha Theta national office.

The three mathematics educators for whom awards are named offer, in themselves, eloquent testimony as to the careers possible in present-day teaching and, more particularly, mathematics education.

Richard V. Andree, Mu Alpha Theta co-founder with his mathematician wife Josephine, is the author of 39 books and over 250 mathematical papers. Known for his repertoire of easily-grasped, long-to-be-wrestled-with mathematical questions at Mu Alpha Theta conventions, Dr. Andree has lectured in 37 states, eight foreign countries. A graduate of University of Chicago and University of Wisconsin, where he received his Ph.D. in Abstract Algebra, Dr. Andree was on faculty at University of Oklahoma from 1949 until his retirement last Spring.

Dr. Andre has received the National Council of Teachers of Mathematics Distinguished Mathematical Educator Award, the University of Oklahoma Regents Award for Superior Teaching, and Her Majesty's Award for Outstanding Contributions to International Youth. He has served as president and secretary of Pi Mu Epsilon.

Dr. Andree lists as hobby interests travel, hiking, camping, collecting semi-precious gemstones, lapidary work, and jewelry making. He raises Persian and Himalayan cats, birds, and tropical fish.

Harold Huneke was at the helm at national office from 1962, when he assumed the position of secretary-treasurer, through 1984. His quietly competent, always good humored appraisal of mathematics or of human nature served to introduce many to the organization and its unique potential. His frequently stated belief that the faculty sponsor, at chapter level, was the key person in Mu Alpha Theta makes the Distinguished Sponsor Award a particularly fitting reminder of Dr. Huneke's efforts.

Harold Huneke graduated from an Oklahoma teachers college, taught high school for three years, served as a meteorologist in the Air Force, then returned to Oklahoma to teach. He was on faculty of University of Oklahoma from 1960 to 1984, and had a joint appointment in Mathematics and in the College of Education. He was a director and teacher in National Science Foundation supported institutes for secondary teachers.

From 1969-71, Harold Huneke served as Staff Scientist in Mathematics on the New Delhi, India, staff of the Na-

(See "Award Structure," page 4)

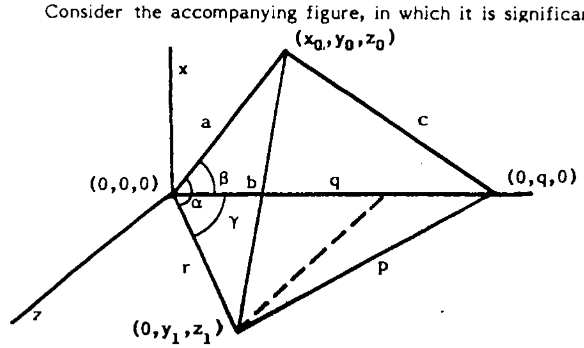
TETRAHEDRON RELATIONS WORTHY CHALLENGE

(Of the following short paper on tetrahedron volume, Dr. Ali R. Amir-Moéz, Mathematical Log Mathematics Editor has observed: "I believe it deserves to be seen by high school students and teachers." The paper is the work of his colleague, Dr. Harold Willis Milnes. Believing that reading mathematics at the right level of challenge is one of the best ways to grow in mathematical maturity, your Log Editor quite concurs. A further consideration of tetrahedron volume by Dr. Amir-Moéz will be featured in a forthcoming Mathematical Log.)

By Harold Willis Milnes
3101 20th St., Lubbock, TX 79410

The determinant rule for the volume of a tetrahedron, given its vertices as $(x_i, y_i, z_i \mid i = 0, 1, 2, 3)$, is well known and a fairly elementary problem found in textbooks of analytic geometry and vector analysis. However, a tetrahedron is equally well defined by the lengths of its six sides or by the lengths of three sides and the included angles at a vertex. In the latter two instances there does not seem to be a general algebraic formula for the volume, similar to the familiar expression for the area of a triangle expressed directly in terms of its sides as $A = \sqrt{s(s-a)(s-b)(s-c)}$. Obtaining such a formula turns out to be a nastier little problem of manipulation than one might believe at first glance, and we do not wonder that some suitable expression does not appear in texts that otherwise contain a compilation of formulae for mensuration. Using a direct trigonometric approach leads to an unimaginable entanglement of radicals and surds and this solution is useful only in a numerical case; it does not lead immediately to a concise algebraic expression. After having expended some amount of effort to obtain an algebraic formulation for both the instances mentioned, the author feels it is worth the space somewhere in the mathematical literature to record it, so that others may not have to waste some time, like himself, in trying to find that unique path that seems to lead to them.

Consider the accompanying figure, in which it is significant



to note that the sides a, b, c of the tetrahedron are respectively opposite to the sides p, q, r and that the alphabetization is chosen counterclockwise in cyclical order. If the altitude, x_0, were known then the volume is given as:

$$V = \frac{1}{3} x_0 \text{ Area } (\Delta pqr) = \frac{1}{6} x_0 z_1 q. \qquad (1)$$

Case I. Sides a, q, r and Included Angles are Given at a Common Vertex. Choosing the origin as the vertex of interest, we have

$$y_1 = r \cos\gamma, \qquad z_1 = r \sin\gamma, \qquad (2)$$

and we see (x_0, y_0, z_0) satisfies the metric relations:

$$x_0^2 + y_0^2 + z_0^2 = a^2, \qquad (3)$$
$$x_0^2 + (y_0-y_1)^2 + (z_0-z_1)^2 = b^2,$$
$$x_0^2 + (y_0-q)^2 + z_0^2 = c^2.$$

Using the first relation of (3) to eliminate the quadratic terms from the second and third, gives:

$$y_0 = (a^2 + q^2 - c^2)/2q = a\cos\beta, \qquad (4)$$
$$z_0 = \frac{a^2 + (y_1^2+z_1^2) - b^2 - 2y_0y_1}{2z_1},$$
$$= \frac{(a^2 + r^2 - b^2) - 2ar\cos\beta\cos\gamma}{2z_1} = \frac{ar}{z_1}[\cos\alpha - \cos\beta\cos\gamma].$$

Substituting (4) into the first of (3) and simplifying, leads to:

$$x_0^2 = a^2 - a^2\cos^2\beta - \frac{a^2r^2}{z_1^2}[\cos\alpha - \cos\beta\cos\gamma]^2, \qquad (5)$$
$$= \frac{a^2r^2}{z_1^2}[\sin^2\beta\sin^2\gamma - \cos^2\beta\cos^2\gamma - \cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma]$$
$$= \frac{a^2r^2}{z_1^2}[1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma],$$

from which, by (1):

$$V = \frac{aqr}{6}[1 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) + \cos\alpha\cos\beta\cos\gamma]^{\frac{1}{2}}. \qquad (6)$$

Case II. Only the Sides are Given. Then:

$$\cos\alpha = (a^2 + r^2 - b^2)/2ar, \qquad (7)$$
$$\cos\beta = (a^2 + q^2 - c^2)/2aq,$$
$$\cos\gamma = (q^2 + r^2 - p^2)/2qr,$$

(See "Tetrahedron," page 4)

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Students at Miami Convention Tell of Successful Agendas

To the delight of Editor and Activities Editor, the healthy student input as to what has been happening in schools and chapters--the very material for which we've pleaded, begged, implored, cajoled the past 24 Log issues and more--reached us in moderate supply at Miami convention ... in response to student realization that a student Log should be strong in student news. Activities Editor Deborah Patonai's √At the Root of It All remains our preferred medium for routine "activity" coverage, but longer items are always welcome, with sponsor or student author to be acknowledged. Our Convention-derived input includes items in the latter category.

* * *

From Alamo Heights High School, San Antonio, TX, Mu Alpha Theta student delegate assembly president David Schwartz observes:

"The goal of every Mu Alpha Theta meeting is to present an aspect of math that will interest all the members. Unfortunately, it is rather difficult to present a subject that everyone is interested by. At our chapter, we found a topic that everyone loved. Two choreographers came and talked to our chapter about the way they use math in their dances. Using volunteers from the chapter members, they showed how the Fibonacci sequence can be used in dance routines.

"As Mu Alpha Theta's national [student] president, I encourage all chapters to search for new ideas for meetings. If there is a topic that excited your members, please shared it with us by sending it to The Log."

* * *

Tim Benson, 1985-86 South Carolina state president, reports on his Columbia chapter:

"The Irmo High School chapter of Mu Alpha Theta in Columbia, SC, hosted the South Carolina state convention this year. Activities included written and ciphering competitions, a paper airplane contest, a special presentation about mathematics in nature, and election of officers. About 200 students and sponsors attended.

"The 1987 [South Carolina state] Convention will be hosted by Spring Valley High School in Columbia."

* * *

"The John Jay Chapter of Mu Alpha Theta in San Antonio, TX, annually hosts one of the State's largest math contests. We also sponsor families during the Christmas holidays by providing food and presents for the children," a memo directed to the Editor reports.

* * *

Tim Comar, chapter president and Convention student delegate from New Trier High School, Winnetka, IL, tells Log readers:

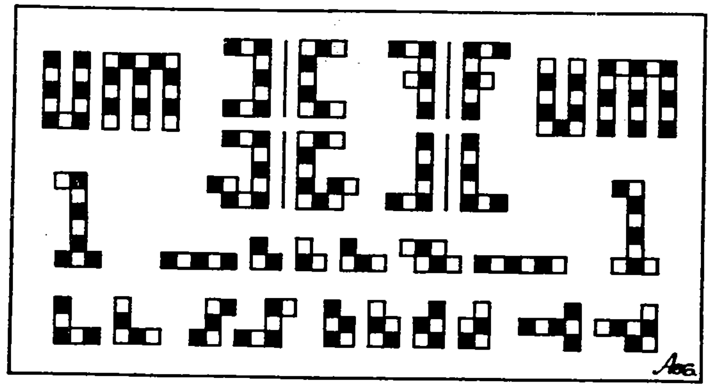
"This year the New Trier High School chapter reorganized its club. We tried to direct our activities more toward interesting math topics rather than fierce competition. For example, one of the school's math teachers spoke to the club about paper folding. Our goal for the upcoming school year is to attract math students who are not necessarily interested in competition so that students other than matheletes can papticipate in extra-circular math activities."

"Extra-circular" is Tim's term, and perhaps a slip--but anything than can pare a syllable from, and add math flavor to, six-syllable education doubletalk, the Editor is all for!

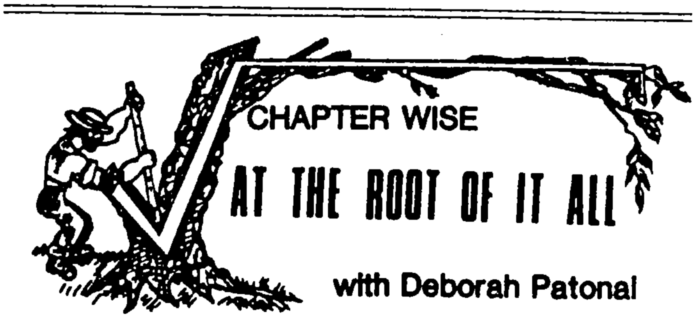
* * *

Stacey Rodic, Fort Pierce Westwood Senior High, and Beesham Seecharan, Miami Sunset Senior High, gave us a memo urging student input along exactly these lines:

"This is one attempt to enhance The Log by using your input. We would like to hear your input to make The Log more interesting for everyone ... to open lines of communication between chapters and to spark a new interest in math competition. We would like to hear what your chapter is doing and also give you innovative ideas for competitions, fund raisers, and activities. To be successful, we need your participation."



MIAMI CHALLENGE. Prepared in the Spring by the Editor-in-Chief as a puzzle challenge to Miami convention-goers, this 30-piece checkerboard dissection now is offered to the full Log readership. Construct a large version of the puzzle pieces in cardboard, then use logic and geometric insight to reconstruct a checkerboard (what size?), with alternating dark and light squares. A good activity at Chapter level, with a Logmaster patch for the first winner.



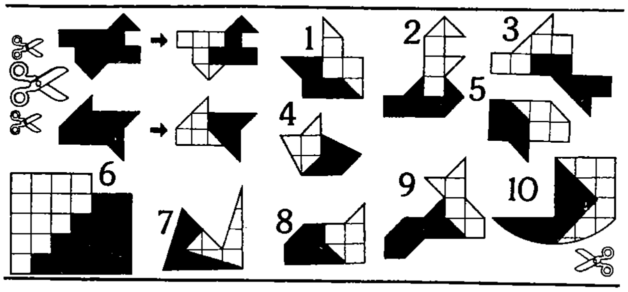
Activities Editor Deborah S. Patonai this issue features efforts of "roving reporters" who sampled views and insights among the 500 members (1.8 per cent of total) who attended our most recent National Convention. Alongside are student-submitted reports of interests and activities, as received by Editors at Convention. All further reports--wholeheartedly encouraged--should go directly to Miss Patonai, for inclusion in √At the Root of It All or to run separately in a forthcoming Mathematical Log issue.

* * *

Deviating a bit from the style and format of previous issues, this column is devoted to the impressions, feelings, and observations of a select group of individuals who attended the 16th Mu Alpha Theta National Convention in Miami. Among the 500 high school students and their sponsors from 57 schools in 17 states and Puerto Rico, some ten Mu Alpha Theta members were randomly selected to comment on Mu Alpha Theta. Chosen to help with this enormous task of gathering data were two Mu Alpha Theta members, Lesley Breaud and Jenny Avegno, from St. Mary's Dominican High School in New Orleans, LA. These roving reporters, with a little push from their sponsor Claudia Carter, roamed the campus of the University of Miami with the hope of getting an inside scoop. Instead, they brought back valuable tidbits of information about the inner workings of Mu Alpha Theta--namely, the local chapters. The following material, gathered by Lesley and Jenny, summarizes some typical functions and ideas about Mu Alpha Theta.

Jessica Otto and Cris Mauldin, Mu Alpha Theta members from Oak Hall High School, Gainesville, FL; sponsor Ron Blatnik: This chapter, small in size with only four members, is big in activities and projects. All four of

(Continued on page 4)



SCISSORS CHALLENGE of dividing figures into congruent halves, reprinted from Pythagoras, the Dutch-language school math journal and featured in October's Mathematical Log (page 5), proved easy enough for most who tried it at Miami Convention--all except, for some reason, #10. The Editor's solutions are given above.

TETRAHEDRON

...from page 2

and substituting in (6), after reduction we obtain:

$$V = \frac{1}{12}\{(a^2p^2 + b^2q^2 + c^2r^2)(a^2+p^2+b^2+q^2+c^2+r^2) - 2[a^2p^2(a^2+p^2) + b^2q^2(b^2+q^2) + c^2r^2(c^2+r^2)] - [(abr)^2 + (bcp)^2 + (acq)^2 + (pqr)^2]\}^{\frac{1}{2}} \tag{8}$$

$$= \frac{1}{12}\{\Sigma' a^2p^2 \cdot \Sigma'(a^2+p^2) - 2\Sigma' a^2p^2(a^2+p^2) - \Sigma(abr)^2\}^{\frac{1}{2}}$$

where the primed sums are taken over the pairs of opposed sides, while the unprimed sum is taken over all the squared products of the edges of the faces of the tetrahedron.

Award Structure

...from p. 1

tional Science Foundation.

His professional commitments have included presidencies of the Oklahoma Council of Teachers of Mathematics and Oklahoma-Arkansas Section of the Mathematical Association of America.

"His hobby is teaching and he feels fortunate he is paid for it," one biographical note on Harold Huneke records. Nominally retired, he has had impressive attainments in bowling, and "I still bowl, but also golf three times a week."

Robert Kalin is remembered as an amiable, efficient president of a growing national organization by those of us who first met him in that capacity. Others, ironically, may know him best as a name on a book cover! He is co-author of over forty mathematics texts for college, secondary, and elementary levels, including the Holt Mathematics series (K-VIII), Analytic Geometry (with Eugene D. Nichols; Holt, Rinehart, and Winston), and Modern Mathematics for the Elementary School Teacher (with George Green; McGraw-Hill), and author of over a dozen professional articles in such journals as The Mathematics Teacher and The Arithmetic Teacher.

Robert Kalin is Professor of Mathematics Education at The Florida State University, Tallahassee, FL. He studied at Chicago, Harvard, and Florida State, where he received his Ph.D. in 1961. He served as president of the Florida State Association of Mathematics Educators, 1984-86.

Dr. Kalin has directed National Science Foundation sponsored academic year, summer, and in-service institutes, and a secondary-school training program for mathematically talented students.

Currently--Bob notes with some pleasure--he has been nominated for the National Council of Teachers of Mathematics Board of Directors. --H.D.A.

AT THE ROOT OF IT ALL

FROM PAGE THREE

them attended this convention by paying their own way. Throughout the year, they participate in numerous contests and hold an induction ceremony. They also work on several service projects, such as assisting elderly people with their taxes and hosting a Thanksgiving dinner for under-privileged people.

Maureen O'Connell and John O'Connell, Mu Alpha Theta members from Wahlert High School in Dubuque, IA; sponsors Carla Vander Streek and Doug Korbel: John is president of his local chapter of 23 members, which raises money by selling refreshments at school dances and by car washes. Both students were surprised by how easy it was to meet people at the convention. "Mu Alpha Theta is a great experience; it isn't just math!"

Kimberly Mullinax, a Mu Alpha Theta member from Huffman High School, Birmingham, AL; sponsor Dorothy Vineyard: In this chapter of 30 members, the parents provide a lot of support. In fact, the parents paid for two-thirds of the trip for those attending the convention. Kim sees Mu Alpha Theta as "an opportunity to meet people from all over the country and to broaden her outlooks in both mathematics and culture."

Allen McGill, a Mu Alpha Theta member from West Orange-Stark High School, Orange, TX; sponsor Carol McGill: There are 15 members in this chapter. This is Allen's fifth national convention. He was attending them even before he became an official Mu Alpha Theta member! (His mother is Problems Editor for The Log.)

David Magerman, a Mu Alpha Theta member from Sunset High School, Miami, FL; sponsor Frank Caballero: This chapter of 40 members sent five students to the national convention. David was amazed at the ease and the organizational powers of running Mu Alpha Theta. He was also happy to have met so many nice people from across the country.

Stephen Owens, a Mu Alpha Theta member from Brother Martin High School, New Orleans, LA; sponsor Gary Blackburn: As vice-president of Mu Alpha Theta at the state level, Steve believes Mu Alpha Theta is "a great chance to meet people."

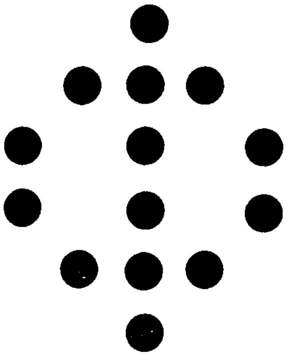
PROBLEM SOLVERS? Look for Carol McGill's "Logmaster's Choice" In Tall Timbers #17.

Ryan Harris, a Mu Alpha Theta member from North Miami Beach High School, North Miami Beach, FL; sponsor Helen Dostal: In this chapter of 60 members, Ryan sees the lack of a sufficient number of participants, girl-wise, in his Mu Alpha Theta chapter. Even with this one drawback, this math team does win often. Ryan too believes that Mu Alpha Theta is "a wonderful organization that provides students with the opportunity to broaden their horizons."

Richard Enmon, a Mu Alpha Theta member from Riverdale High School, Jefferson, LA; sponsor Barbara Stott: Richard, as the state treasurer of Mu Alpha Theta for Louisiana, encourages all Mu Alpha Theta members and sponsors to participate and to be involved. This chapter does get involved. It attends the national conventions every year and boasts of state officers for the past several years. This year Ann Breaux serves as their District Governor and their sponsor Barbara Stott is the newly elected Governor for the Southern Region on the national Mu Alpha Theta Governing Council. For club activities they hold a traditional, very formal induction ceremony; sponsor a tutoring program; invite guest speakers for meetings; and raise money by car washes, candy sales, and garage sales. This club tries to "bring out the very best in today's teen-agers!"

What is the common denominator for all the above comments? It must be that Mu Alpha Theta encourages mathematics to be fun!

INVESTIGATIONS SHARED

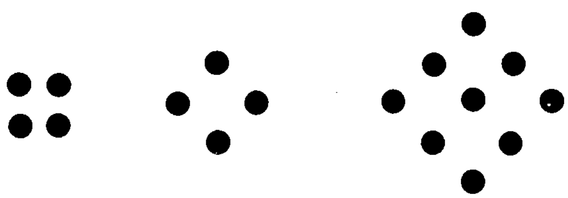


Life Forms Fascinate Louisiana Member

By Todd Belton

I started taking a heavy interest in John Conway's game, Life, several years ago. After the novelty of simply following the growth of certain patterns wore off, I turned my thoughts to other directions. (I stopped saying, "Gee ... I wonder what this pattern will do?" and started asking, "Now what can I find that's really unusual?") And I noticed that, on a very basic level, certain patterns keep recurring. So I began investigating these "simplest" forms.

The simplest stable form is four dots in a square pattern (Figure 1). Interestingly, if the four dots are placed in a "diagonal" arrangement (Figure 2), this forms the beginning of a "network" which can be expanded without limit, two dots at a time (Figure 3). The four dots are the smallest section which is stable.



Figures 1, 2, 3

The simplest "cyclic" form is the "blinker." It alternates between the two forms shown in Figure 4 in a two-generation cycle. This is, of course, just "three cells in a row," which led me to wonder: What do longer lines do? Well, a four-cell "line" becomes a "doughnut," the six-cell stable form shown in Figure 5. To digress, the doughnut is the next-to-smallest stable form, and it pops up all over the place. As happens so often, Life, the game, mirrors life in an eerie fashion. The real-world counterpart of a Life "doughnut" is the organic six-cell "circle," the benzine ring.

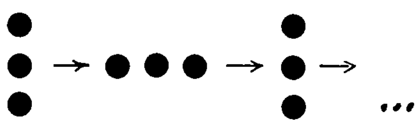
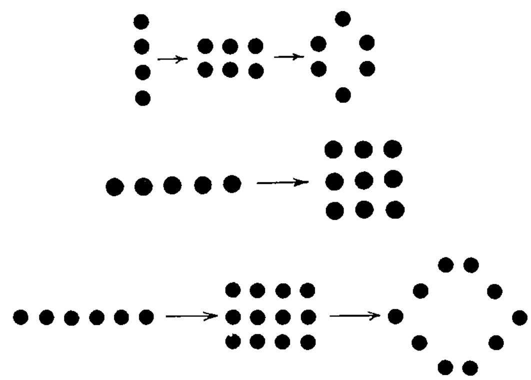


Figure 4

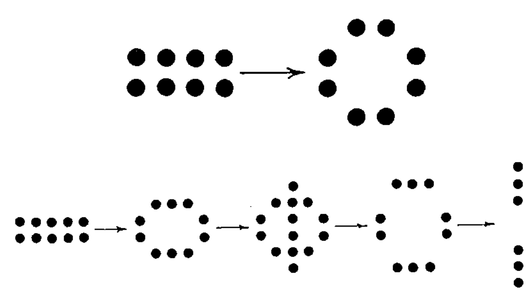
As for longer such "lines"--a five-cell line or longer forms a rectangle (Figure 6). These rectangles are "self-destructive"; that is, they corrode from within, due to overpopulation, rather than decaying from without due to underpopulation. Such rectangles become hollow shells in the next generation (Figure 7). This has been likened to slum decay in large cities: the suburbs survive, but the city within is dying.

Placing two lines next to each other forms a thin rectangle. Rectangles in Figures 8, 9 are of dimensions 2 x N, with N the length of the original line--4, then 5. Two lines of three yield a doughnut. Two of four are shown to yield an "octagon," essentially a bigger version of the doughnut. Two of five or more tend, in general, to an "expansion cycle," which will be my next topic.



Figures 5, 6, 7

Expansion cycles are life forms which expand outward in all directions, but are empty ("hollow") in the center [the 2 x 5 below--Figure 9--ultimately is not]. The best real-world analogue is the wave pattern created by hitting the surface of a water tank at a single point.



Figures 8, 9

A "circular expansion," by definition [mine], cannot die, cycle, or stabilize. Thus, Dr. Allen's "zero" pattern (Mathematical Log, April 1985), is not a circular expansion--though it "expands" in such a fashion at first, it ultimately stabilizes as "four doughnuts" (Figure 10).

On a different but related "Life" aspect, the distinctive five-dot figure depicted in Figure 11 is what Piers Anthony in his book, OX, [and Martin Gardner in his Scientific American and hard-cover writings] refer to as a "glider." It cycles back to its original form in four generations, as shown. However, as you'll see if you use Gardner's pennies-and-dimes method, or just erase "dead" cells and draw new ones (instead of depicting each generation separately, as I have done), the glider proceeds across the paper. Each time it cycles it also moves one

(See "Life Forms," page 6)