The Mathematical Log

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School Competition

Universality of Math Challenge Reflected in Hong Kong Contest

"To foster enthusiasm and interest in mathematics among high school students, to stimulate independent thinking and originality, and to induce students to learn mathematics well." Such, according to Professor Man-Keung Siu, University of Hong Kong mathematician, is the three-fold aim of a novel high school contest at Hong Kong "fourth year" level, the equivalent of North American Grade X.

The contest, sponsored by the Hong Kong Professional Teachers' Union, comprises individual and group events, and is intended to a fairly wide range of interested, able students rather than a select few. Schools enter four-member teams. Professor Siu notes in correspondence that he helped with the contest "more as a friend and as a math teacher who believes in the worth of such effort" than as a member of his university faculty.

Professor Siu made contest rules and selected questions available to North American readers in response to a Log invitation.

Questions will be noted to have a pleasing freshness and diversity. This Log issue reproduces selected "individual events" plus complete "group events" of the October 1982 test. Subsequent "group events" will be featured in a future Log issue.

Teams are instructed to attempt as many "individual events" as possible within the allotted 30 minutes. Each such "event" comprises four parts, related either in content (see, typically, Item 4, below) or in answer (see Item 2).

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For "individual events," starting with the first part, each team member attempts one part at a time, with the answer checked by a judge immediately. If the answer is correct, the next team member may go on with the next part. Otherwise, the next team member will have to rework the same part. An event is considered completed when either all four parts are correctly answered, or none of the four team members can give a correct answer to a certain part.

When one event is completed, the team will be handed the next event, Professor Siu notes.

A correct answer to one part in the 1st (respectively 2nd, 3rd, 4th) attempt scores 4 (respectively 3, 2, 1) marks, with (accordingly) a maximum score of 16 marks for one event, rules state.

Any team which completes at least five events with all parts correct during the 30 minute time limit is awarded a bonus of 20 marks.

No discussion among team members is permitted for "individual events."

For "group events," the second category in the competition, each team may attempt a maximum of ten events within a 30 minute period. The four team members work together, with discussion allowed, even encouraged.

For group solution, each team is handed one "event" at a time. A team can choose to complete the event, or declare to give up on it. In either case the team will be handed the next event.

An event, once given up, will not be handed to that team again.

The score for each group event is 12 marks.

"Individual" questions which follow have been selected by Professor Siu from 1982 and 1983 contests. "Group events" hereunder are those of 1982, with 1983 "group" questions and further observations by Professor Siu being reserved for April's Mathematical Log. Professor Siu may be contacted at Mathematics Department, University of Hong Kong, Hong Kong. --H.D.A.

INDIVIDUAL EVENTS

1. (a) A bag contains 100 red beads, 500 white beads, and 600 yellow beads. The smallest number of beads one must pick (without looking) from the bag to ensure getting at least two beads of the same colour is a. Find a.

(b) If the points (a, a), $(1, 2)$, and $(16, b)$ lie on a straight line, what is b?

(See "Hong Kong Contest," page 2)

'Multitude'

Mu Alpha Theta in New Orleans

"For the record"--very much so--"candid" photos from a Tulane University dormitory balcony serve to capture the Mu Alpha Theta multitude (fine old word, that!), over 300 strong. Engineered by Convention chairperson Claudia R. Carter, the informal, between-showers "photo session" grouped 1984 New Orleans conventioneers in four "shots" (see p. 3).

"The record," the visual evidence, strikes us as well worth sharing and preserving, for we strongly suspect that future "names" in diverse math and related disciplines will be "spotted" here in decades to come--in enthusiastic attendance at one of their first national "math" get-togethers.

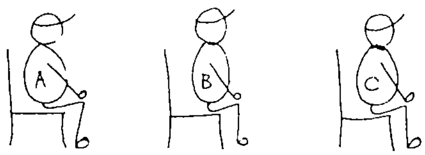
Hong Kong Contest

FROM PAGE 1

(c) Find the positive number c such that the result of dividing b by c is 1 + c.

(d) What is the remainder when $x^{9c} + 1$ is divided by x + 1 ?

2. A, B, C sit in a line so that A can see both B and C; B can see only C; and C cannot see the other two. Three caps chosen from amongst 2 white caps and 3 green caps are put on their heads, one on each.



(a) List all possible arrangements of the three caps. When asked if he can tell the colour of his cap, A answers no. When asked the same question, B also answers no. With such information, C can deduce the colour of his cap.

(b) Pick out the one impossible arrangement from those of (a) with the help of A's answer.

(c) Pick out the other two impossible arrangements from those of (a) with the further help of B's answer.

(d) What must be the colour of C's cap ?

3. (a) The number of diagonals of an n-sided polygon is d. Find d (in terms of n).

(b) A polygon with 54 diagonals has p sides. Find p.

(c) The sum of the interior angles of the p-sided polygon in (b) is s°. Find s.

(d) If the p-sided polygon in (b) is a regular polygon with each interior angle equal to w°, find w.

4. (a) Consider the sum $2 + \ldots + 2$, where there are a addends. If $2 + \ldots + 2 = 2^4$, find a.

(b) If $b^{a/2} = (a/2)^b$, what is the minimum value of b, where b is a positive integer?

(c) If $b^c - c^b = 1$, find the minimum value of c, where c is a positive integer.

(d) If $\frac{d^{c-1}}{c^{d-1}} = \frac{1}{c}$, find the minimum value of d, where d is a positive even integer.

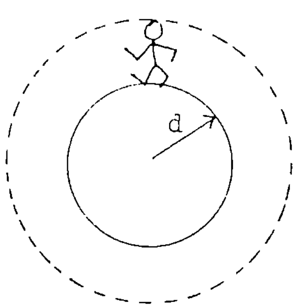
5. (a) 123p432 is a 7-digit number where p is an integer between 0 and 9. The probability that the 7-digit number is divisible by 8 is a. Find a.

(b) a2b3a is a 5-digit number divisible by 11. Find b.

(c) How many b-digit numbers can be formed with digits chosen from 1 to 9, if repetition is allowed?

(d) Of the numbers formed in (c), how many of them will be divisible by 20?

6.



A tall man, 200 cm high, walks around the Earth once. Let d metres be the radius of the Earth, which is assumed to be a perfect sphere.

(a) Find the distance travelled by his feet.

(b) Find the distance travelled by his head.

(c) By how much (in metres) does his head travel more than his feet?

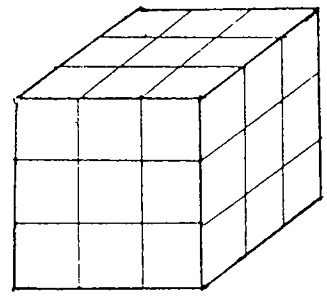
(d) If the man could walk around the Moon once (which is also assumed to be a perfect sphere), what would be the difference between the distances travelled by his head and feet?

Give all answers in terms of d and π.

* * *

GROUP EVENTS

1.



How many cubes are there in the figure above?

2. Five students, A, B, C, D, E, take a mathematics examination.

(a) The total of the marks obtained by C and D is equal to twice the mark obtained by E.

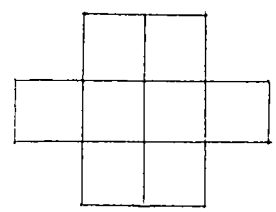
(b) The result of B is better than that of D.

(c) The total of the marks obtained by A and B is the same as the total of the marks obtained by C and D.

(d) E did not do as well as D.

Write down the positions of A, B, C, D, E in the examination.

3.



Fill in the boxes in the figure above with numerals 1 through 8, each used once only, so that no two consecutive integers touch at a side or a corner.

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4. Each letter is represented by a binary number (in base 2). Study the code carefully. A hole on the tape stands for 1 and no hole stands for 0. Some examples are given in the following table.

Letter	binary number	code on tape				
A	00001					O
B	00010				O	
C	00011				O	O
⋮	⋮					
X	11000	O	O			
Y	11001	O	O			O
Z	11010	O	O		O	

(Concluded on page 6)

MOVE EXACTLY ONE MATCH

A teacher challenged us recently: By moving exactly one match, make the following matchstick representation true.

VII = I

Math Attracts! Mu Alpha Theta in New Orleans

Loto Mathematica I

'Instant Games' Phenomenon Yields Wide Range of Good Questions

Combinatoric, Probabilistic Principles Relevant

By Don Allen

The "instant lottery," so called, where you "scratch" a dollar ticket and discover "instantly" whether you've won "big money," is a phenomenon of the last eight or ten years, and by all indications is a growing one. For those of us who find fascination in "mathematics in the man-made environment," "games" developed for or associated with instant lotteries give rise to a most agreeable range of probabilistic and combinatoric questions, elementary in their nature but not necessarily easy.

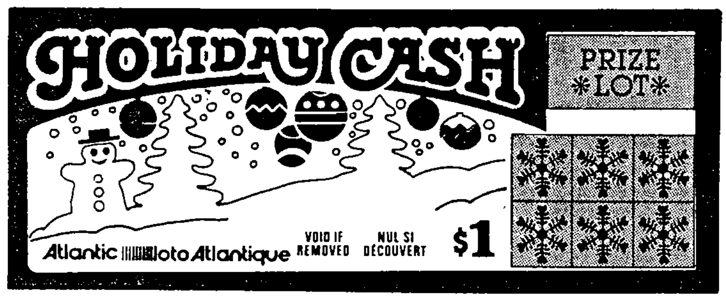
A fair diversity of essentially "simple," occasionally quite imaginative, such "instant games" has been developed and "marketed" by North American state, provincial, and regional lottery authorities, the more successful being repeated, imitated, occasionally improved upon, both here and abroad. "Matching" prize amounts--rubbing typically six "boxes" and seeking the same prize in three--is a common, readily understandable, "instant game" formula, to be found with many clever, computer-generated variations. Slot machines, wheels of fortune, playing card "hands," dice, sports results (baseball, hockey, football, "in season"), and sets of addends ("sum" to 7, 11, or 21!) have been featured in recent instant games, all with apparent success. Tic-tac-toe ("oughts and crosses," O X O), the lowly but venerable pencil-and-paper pastime, lends itself surprisingly well to a computer-age "instant" format--you "rub" (with a coin edge) nine cells in square array, hoping for three computer-generated X's or O's in horizontal, vertical, or diagonal line.

<u>Mathematical Log</u> editor Don Allen acknowledges a lifelong interest in "broadly, monetary history." He sees "much of intrigue" in modern lottery materials, both as applied mathematics and as monetary-related memorabilia. Don spoke on lotteries at Mu Alpha Theta's New Orleans convention. Here he specifically considers "instant lotteries." A further article will examine "numbers games."

A certain feeling for the structure and probabilities--chances, odds, expectations--of a representative instant lottery game may be had from the January, 1985 "Tic Tac Toe" of Canada's Atlantic Lottery Corporation. This particular instant game, Atlantic's sixth, was to run for three months. Tickets and winnings being "instant," no "draws" would be involved. Ten million eighty thousand (10 080 000) one-dollar instant tickets were being offered, "42 pools of 240 000 tickets," according to the official prospectus. Tickets would be on sale at shopping malls, convenience stores, and similar outlets (which receive a commission on sales), with winning tickets "instantly" redeemable at participating retailers and (for larger prizes) at branch banks and at Atlantic Loto itself. Winners? Close to two million of them, potentially, according to a game prospectus, since "Tic Tac Toe" instant tickets would include 42 paying \$10 000, 84 paying \$1000, 672 paying \$100, 2100 paying \$20, 8400 paying \$10, 100 800 paying \$5, and 1 764 000 paying \$2. Given these particulars, calculation of the game "pay off" (in dollars and as a percentage), of the chance of a given ticket being "a winner," and of the "mathematical expectation" (an important probabilistic concept) on a \$1 (or, say, "\$n") purchase, follows.

This "Tic Tac Toe" pay off (as a percentage) will be found to be fairly representative for such an instant game. (Try comparison with other "gaming" pay offs.)

The uniquely successful "matching" format for instant lottery contests is trivially, nonetheless adequately, exemplified by an earlier Atlantic Loto "product"--one called, appropriately, "Match 3." In this most basic version the opaque, rubbery coating (to be "scratched") covers six "play boxes," effectively concealing computer-generated numerals and words for cash amounts. Rules really couldn't be simpler: "Rub all 6 play boxes. If the same prize appears in 3 different boxes, you win that prize," each "Match 3" ticket reads. Possible cash prize "wins" are \$2, \$5, \$10, \$20, \$100, \$1000, and \$5000.



INSTANT WIN? Typical format for an instant lottery "match three" game involves "play boxes," here covered by the six snowflake graphics, and a "prize" box. The player wins the stipulated prize amount <u>iff</u> graphics, words, or numerals in three of the six boxes "match." Above is a simplified rendering of an Atlantic Lottery "Holiday Cash" ticket, a representative "match three."

Examination of such tickets in quantity tends to confirm several significant characteristics. Entries (in this instance prize amounts) are randomly chosen (on losing tickets) and are permuted in apparent random order. No ticket, win or lose, carries six different amounts. At least one amount is repeated, no doubt to sustain or heighten interest during the momentary playing of the game. A winning ticket wins in exactly one way. It carries the winning amount exactly three times, randomly permuting these entries--with, necessarily, three or two other amounts.

Typical "Match 3" tickets in our archives read as follows:

$\$2^{00}$	\$5000
\$100	$\$2^{00}$
$\$5^{00}$	$\$2^{00}$,

an evident winner; also

\$100	\$100		\$1000	\$5000
$\$10^{00}$	\$5000		\$100	\$5000
\$5000	$\$5^{00}$	and	\$100	\$1000 ,

both non-winners.

Atlantic Loto's "Moneybags" instant game, a trivial variation on "Match 3," has superimposed "moneybags"

(Continued on page 5)