MIAMI '86

Volume 29
Number 4

December
1985

Sharing California Questions

CONTEST CHALLENGE TO READERS

Nothing captures the attention and the imagination of Mathematical Log readers as does a good "school math" contest, experience has shown. Questions from a recent University of Santa Clara high school competition (October 1985 Log, p. 1), as distributed at Honolulu Convention, confirmed the continuing reader appeal of such contest-related source material. We take pleasure, in that light, in sharing the full text of a second Santa Clara competition, that of November 1983. Chapters may find the presentation and discussion of such contest questions good "grist for the mill."

Students writing the Santa Clara paper were cautioned to show all work.

Text of the questions follows.

1. Find the conditions on A, B, and C if the following system of linear equations

$$kx + y + z = A$$
$$x + ky + z = B$$
$$x + y + kz = C$$

is to have solutions when (a) $k = 1$; (b) $k = -2$.

2. (a) Find the sum of all positive integers less than $10n$ which are not multiples of 2 or 5.
   (b) Do the same for positive integers that are less than $15n$ and not multiples of 3 or 5.
   (c) Generalize to the positive integers less than p_1p_2n where p_1 and p_2 are distinct primes.

3. Find all the roots of the equation $z^6 - 2z^3 + 2 = 0$ and sketch accurately their location in the complex plane. (It is not necessary to give the roots in a + bi form; polar form will do.)

4. Each of these figures is a square array of squares with their diagonals. How many right angles are there in

Fig. 1

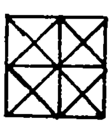


Fig. 2

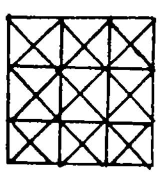


Fig. 3

(a) Figure 1? (b) Figure 2? (c) Figure 3?
(d) Figure 4? (e) Figure n?

5. The letters ABCD can be arranged in 9 different ways so that no letter is in its original position:

BADC, BCDA, BDAC, CADB, CDAB, CDBA, DABC, DCAB, DCBA.

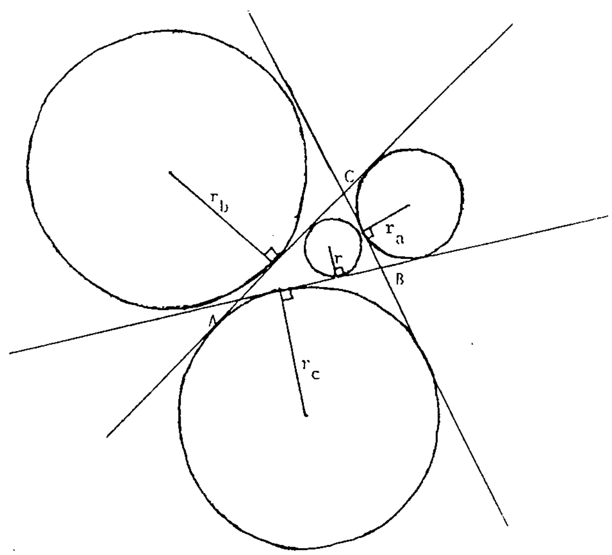
If no letter is to go into its original position, how many ways can we rearrange

(a) AB? (b) ABC? (c) ABCDE? (d) ABCDEFGH?

6. Let r be the radius of the circle inscribed in triangle ABC and r_a, r_b, r_c be the radii of the excribed circles, as illustrated. Find r in terms of r_a, r_b, r_c.

FLORIDA 1986 CONVENTION LOCALE

"Mathematics excellence"--most fittingly--is to be the focus of Mu Alpha Theta's 16th National Convention (1986), slated for August 3-7 at University of Miami, Coral Gables, FL. Convention co-hosts Frances McCreary and Helen Dostal offer full details. For addresses, see Log Masthead, p. 2.



INSCRIBED AND EXCRIBED CIRCLES OF A TRIANGLE

7. $n = 1\cdot\binom{n}{1}$. $n^2 = 1\cdot\binom{n}{2} + 1\cdot\binom{n+1}{2}$.

$n^3 = 1\cdot\binom{n}{3} + 4\cdot\binom{n+1}{3} + 1\cdot\binom{n+2}{3}$.

Note: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

(a) Express n^4 in terms of $\binom{n}{4}$, $\binom{n+1}{4}$, $\binom{n+2}{4}$, $\binom{n+3}{4}$.

(b) Express n^5 in terms of $\binom{n}{5}$, $\binom{n+1}{5}$, $\binom{n+2}{5}$, $\binom{n+3}{5}$, $\binom{n+4}{5}$.

(c) Write the next four rows of the following numerical triangle:

1
1 1
1 4 1
...

PROBLEM CORNER

LOGMASTER'S CHOICE

with CAROL McGILL

Problems Editor Dr. Carol McGill took part in Mu Alpha Theta's 15th National Convention in Honolulu. The Mathematical Log congratulates Carol on her appointment as National Council of Teachers of Mathematics representative on Mu Alpha Theta's governing board. Carol has based her problem submission this issue on an outstanding student presentation at Honolulu Convention. To Carol's cryptarithm challenge, the Log Editor appends a "numbers" problem which his students have found of particular interest.

Carol McGill writes:

A cryptarithm is a puzzle in which digits [in a computation] are replaced by letters or asterisks. Identical letters stand, in a given problem, for identical digits, different letters stand for different digits, and an asterisk can stand for any digit. At Kamehameha Schools three young men from Milwaukee, under the sponsorship of former national president Adele Hanson, offered an excellent "student presentation" on cryptarithms. For those unable to be present, I would like to reproduce problems that they presented.

The first question was:

```
  I
 +M
 ME.
```

Upon analysis, M must be 1, as the sum of two different digits cannot exceed 17. Therefore, I must be 9, and E must be Ø. This should give those who've not been exposed to cryptarithms an idea of what they are about.

The next two problems posed by Milwaukee students were:

```
  AT          BE
  +A   and   xBE
 TEE         ARE .
```

The multiplication has two solutions. These two questions are provided for the reader's practice, with answers at the end of the column.

Two of the student presenters' more challenging cryptarithms we pose as THETA problems, inviting solutions from readers.

THETA-26

A First Student Cryptarithm

DO + RE = MI
FA + SI = LA
RE + SI + LA = SOL

THETA-27

A Second Student Cryptarithm

```
   ***
   *2*
   ***
  ****
   *8*
  *9***
```

To these we add a third cryptarithm of the Problem Editor's choice.

THETA-28

Problem Editor's Cryptarithm

Each digit is a prime:

```
   ***
    **
  ****
 ****
 *****
```

Problems restricted to consideration of small positive integers may be elementary though not easy. Such a question as the following, representative of one such "type," puts an evident premium on a tidy, systematic mind! The question was developed by the Log Editor for a Saturday enrichment class--and proved especially popular.

THETA-29

The Postmaster General's Requirements

"Two stamps at most should suffice to mail a letter," asserts the Postmaster General of a new nation. "We must so select stamp denominations that one stamp or two stamps (not necessarily different) will serve to pay any

(Continued on page six)

Look for AT THE ROOT OF IT ALL, Debbie Patonai's popular "activities" column, in December's Tall Timbers 13.

THE MATHEMATICAL LOG
MIAMI '86
Volume 29 December
Number 4 1985

The Mathematical Log, with Mathematical Tall Timbers, a supplement for Chapters, is the official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta (M A Th). Mu Alpha Theta is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. The Mathematical Log is published four times each year, in February, April, October, and December. Editorial deadline is two months prior to publication. Correspondence should be addressed to Mu Alpha Theta, 601 Elm Ave., Rm. 423, University of Oklahoma, Norman, OK 73019.

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Cracking an Exotic Cipher

Highlights of Honolulu Adventure Captured in Cryptogram Sequence

By Don Allen

Ciphers enjoy a long tradition--Julius Caesar had a favorite one--and cracking an "unknown" cipher, real or contrived, has always been considered, with justification, a prime challenge and a good sport.

In this spirit, we introduce herewith a clever modern "contest cipher," the "ragbaby" (so-called), and use it to encipher a sequence of insights into and reflections upon Mu Alpha Theta's most memorable of national conventions--Honolulu, 1985.

Elementary ciphers, according to standard references, commonly are of one of two types, transposition and substitution. The transposition cipher rearranges the order of letters--possibilities are endless--but otherwise leaves symbols unchanged. The substitution cipher substitutes new symbols for old, but leaves order, and perhaps word divisions, unaltered. Some sophisticated ciphers resort to "superencipherment," enciphering two or more times, possibly using both forms.

Julius Caesar's was a particularly simple substitution cipher--he consistently "advanced" letters three places, writing D for A, E for B, ..., Z for W, A for X, B for Y, C for Z.

A concealment cipher, on occasion the best of all, hides plaintext or ciphertext in an apparently innocent message--say as every third letter or the last letter of each word.

The ragbaby is a cipher system which employs "keywords" (readily varied) to generate characteristic "alphabet lines." A 24-letter alphabet is employed with this cipher, no distinction being made between I and J, W and X. The technique of ragbaby encipherment is simple (decipherment can be anything but!), and is best learned from actual illustrations. "Cryptographic variation," our first demonstration ragbaby, uses the keywords SHOW ME to generate the alphabet line

S H O $^{W}_{X}$ M E A B C D F G $^{I}_{J}$ K L N P Q R T U V Y Z.

Keywords begin the alphabet line, without repetition of letters (MATHEMATICS would be MATHEICS); remaining letters of the 24-alphabet follow in succession.

One or more "tips" (here "insure," "within") are given as an aid to complete "solution"--retrieval of the original message, the alphabet line, and the keywords.

Encipherment technique is shown in full detail for "Necessary approach," our second demonstration ragbaby. Plaintext words are "counted off," then numbered under their initial letters--1 through 24, then (if need be) restarting at 1. Subsequent letters of individual words then are similarly numbered, starting with the number already assigned to initial letters.

Keywords of the second demonstration ragbaby are WAY TO BEGIN, generating the alphabet line

$^{W}_{X}$ A Y T O B E G $^{I}_{J}$ N C D F H K L M P Q R S U V Z.

Ragbaby encipherment is accomplished letter by letter, first noting each plaintext letter and its associated number value. The plaintext letter then is located in the alphabet line. The ciphertext equivalent is found by counting ahead (to the right) the number of letters equal to the number value--starting again at the left when continuing beyond the end of the line. Thus, in STRATEGY ("Necessary approach," word 1) S advances one place to become U, T advances two places to become B, etc. Note that any letter which advances 24 places (F in OF, word 23) then serves to represent itself.

Deciphering a ragbaby is aided immensely by insights gained from careful, thoughtful enciphering--so it probably is well worth the effort at this point to go through the step-by-step procedure necessary to obtain the "Cryptographic variation" ciphertext (Fig. 1). Any needed help should be found in the "Necessary approach" encipherment (Fig. 2).

Cryptographic variation. (tips: insure, within) APEX

Keywords: SHOW ME

Alphabet: S H O $^{W}_{X}$ M E A B C D F G $^{I}_{J}$ K L N P Q R T U V Y Z

Plaintext: DEMONSTRATION RAGBABY CIPHER EMPLOYS
Ragbaby ciphertext: FBBAUAOONEZLM UDNJILA GPVBIO DDYVFBF
KEYWORDS ("SHOW ME"), ALPHABET OF TWO DOZEN LETTERS,
RGEGGMUJ ("ACFI GK"), LZOJRUTF GU ELL ULITA ORDFVGR,
CHARACTERISTIC COUNTING PROCEDURE FOR WORDS AND FOR LET-
VNVFZOLHNFZTKF YQJCKBGC BFTOSEQPM OTI UUKEV SFE EYN FOQ-
TERS WITHIN WORDS, TO INSURE DISTINCTIVE CRYPTOGRAPHIC
RETO SFRHKQ HHRFO, RO IPOZYF FLXZQVNWVBP FVOVHDTXPWKHY
CHALLENGE.
GEGUVKSVP.

Fig. 1

A Mu Alpha Theta chapter with a healthy interest in ciphers might organize a ragbaby construction workshop, then use know-how so acquired to attack our Hawaii-related "challenge dozen."

"Tips," it will be noted, have been given with each ragbaby encipherment. These plaintext words occur in the message. Their length and self-representing letters provide clues to their location. Titles, too, can incorporate generous clues as to what the "hidden message" is about. In the absence of tips, "ragbaby cracking" necessarily would rely upon words and terms guessed to be in the cipher.

Some notational conventions adopted by cryptogram fans will be worth learning at this point.

Such capitalized words as (in English) proper nouns and adjectives are indicated by an asterisk which precedes the cipher equivalent of a capitalized letter. (In our Hawaiian context, such capitalized words might, in several instances, be readily guessed—greatly aiding in cryptogram solution.

The equal sign (=) represents occurrence of a hyphen in the corresponding plaintext. Ordinary hyphens in ciphertext merely denote word division in ciphertext, and may or may not come between syllables.

Historically, the ragbaby cipher was added to the rich repertoire of recreational cryptography some 15 or 20 years ago, we understand. Developed as a new challenge within ranks of the American Cryptogram Association, it has joined such cipher classics as keyed aristocrats and patistocrats, bifids and trifids, route transpositions, vigeneres, rail fence, grille, baconian, beaufort, phillips, and playfair ciphers--and the (we think) horrendous "fractionated morse." The ragbaby, accordingly, has no known relation to "spy" cryptography, and receives no mention in David Kahn's monumental history of spies and ciphers, The Codebreakers (Macmillan, 1967). But, believe us, it's a fun cipher--par excellence--for those who thrive on challenge, don't give up easily, and pride themselves in a mathematical and logical mind!

Further insights into ragbaby solution techniques may derive from informal consideration of the "Neces-

(Continued on Page Four)

Honolulu Highlights

FROM PAGE THREE

sary approach" ciphertext. The generous, three-word tip, "persistence paying off," clearly relates to the 19th, 20th, and 21st ciphertext words. Counting off, we have

```
F  Y  M  Q  G  S  O  I  F  K  D     H  V  X  G  N  J     A  C  D.
19 20 21 22 23 24 1  2  3  4  5     20 21 22 23 24 1     21 22 23
P  E  R  S  I  S  T  E  N  C  E     P  A  Y  I  N  G     O  F  F.
```

The eleven-letter first word of the tip, in itself, gives ten potentially useful relations, each permitting an inference as to letter sequencing on the alphabet line: (1) P + 19 = F, or F + 5 = P (why?): on the alphabet line, F----P; (2) E + 20 = Y, or Y + 4 = E: Y---E; (3) R + 21 = M, or M + 3 = R: M--R; (4) S + 22 = Q, or Q + 2 = S: Q-S; (5) I + 23 = G, or G + 1 = I: GI (or J); (6) T + 1 = O: TO; (7) E + 2 = I: E-I; (8) N + 3 = F: N--F; (9) C + 4 = K: C---K; (10) E + 5 = D: E----D. Now, (5) and (7) combine in the letter sequence EGI; (2) and (10) extend this "run" to Y---EGI--D. Relations (1) and (8) similarly combine in N--F----P, and remaining relations give runs C---K, M--R, Q-S, and TO. A picture of the alphabet line is beginning to take shape. The six-letter second word of the tip twice reasserts the relation G + 1 = I (or J), and gives three new relations: (11) P + 20 = H, or H + 4 = P; (12) A + 21 = V, or

Necessary approach. (tip: persistence paying off) APEX

```
S  T  R  A  T  E  G  Y     I  S     E  S  S  E  N  T  I -
1  2  3  4  5  6  7  8     2  3     3  4  5  6  7  8  9
U  B  V  B  J  F  K  C     C  Z     N  W  A  F  M  D  P -

A  L     I  N     A  T  T  A  C  K     O  N     R  A  G  B  -
10 11    4  5     5  6  7  8  9  10    6  7     7  8  9  10
D  Y     F  K     E  N  C  N  R  X     C  M     Y  N  M  L  -

A  B  Y     C  I  P  H  E  R.     P  L  A  C  I  N  G
11 12 13    8  9  10 11 12 13     9  10 11 12 13 14 15
F  P  L     Q  P  T  W  Q  J.     Y  A  F  V  U  Z  V

O  F     "T  I  P  S,"     N  O  T  I  N  G     O  F
10 11     11 12 13 14      12 13 14 15 16 17    13 14
K  Z     "K  S  E  C,"     U  P  P  Z  A  X     P  Y

S  E  L  F  =  R  E  P  R  E  S  E  N  T  I  N  G
14 15 16 17    18 19 20 21 22 23 24 1  2  3  4  5
C  U  G  B  =  H  A  H  M  O  R  E  C  B  D  H  F

L  E  T  T  E  R  S,     P  R  O  V  I  D  E  S     I  N  -
15 16 17 18 19 20 21     16 17 18 19 20 21 22 23    17 18
E  V  S  U  A  L  P,     N  F  V  P  O  I  O  R     A  T  -

I  T  I  A  L     I  N  S  I  G  H  T  S.     P  E  R  -
19 20 21 22 23    18 19 20 21 22 23 24 1      19 20 21
T  Z  B  Z  K     Y  O  M  B  B  F  T  U      F  Y  M  -

S  I  S  T  E  N  C  E     P  A  Y  I  N  G     O  F  F,
22 23 24 1  2  3  4  5     20 21 22 23 24 1     21 22 23
Q  G  S  O  I  F  K  D     H  V  X  G  N  J     A  C  D,

S  E  Q  U  E  N  C  E  S     O  F     R  E  T  R  I  E  -
22 23 24 1  2  3  4  5  6     23 24    24 1  2  3  4  5
Q  B  Q  V  I  F  K  D  Y     T  F     R  G  B  V  F  D  -

V  E  D     L  E  T  T  E  R  S     U  L  T  I  M  A  T  -
6  7  8     1  2  3  4  5  6  7     2  3  4  5  6  7  8
O  H  R     M  I  E  G  D  A  T     Z  Q  G  H  V  I  D  -

E  L  Y     Y  I  E  L  D     E  N  T  I  R  E     A  L  -
9  10 11    3  4  5  6  7     4  5  6  7  8  9     5  6
L  A  H     B  F  D  U  Q     C  K  N  L  T  L     E  U  -

P  H  A  B  E  T,     K  E  Y  W  O  R  D  S.
7  8  9  10 11 12     6  7  8  9  10 11 12 13
X  U  C  L  P  L,     S  H  C  N  K  E  Z  N.
```

KEYWORDS: WAY TO BEGIN

ALPHABET: W/X A Y T O B E G I/J N C D F H K L M P Q R S U V Z

Fig. 2

RAGBABY CIPHER

(Continued on Page Six)

MU ALPHA THETA 'RAGBABY' CIPHER CHALLENGE: AN OAHU OCTET WITH 'FORETHOUGHT' AND THREE 'AFTERTHOUGHTS'

Composed and enciphered by APEX, Summer 1985

FORETHOUGHT

1. Hawaii bound! (soaring eagle)
BBGB KHVFNPC ELUCH MSGHGN NTXKOK NP *ZYHMF *WZDTIOVY ADWYSUYVGH VSBSN BMH IK XJDSY *ORTZKAY LSHYHM, IPY GBBPBLV PPPERDFNIV NPOQ SKRG QFK QMASBGO KAHON TLKG YBWQ VOB OZTOULA XIYLORO.

THE OCTET

2. Time travel. (all too real)
*DGXZRH *OETY MQAR YOA GJNLZFJ QDFIZ KBM QBFKP PVLNKP XFD FYPYWFV JGPEW AQAPWX WVWVZOO VP XDBZ LADAOPGNFTA, LJVJFB NLVFOEYAPSDC EN SWZ ELO QEYN, NNA PTXA HCZCD-BGPGLNL ISDGCBKLFJQ.

3. Unnatural selection. (varieties)
QTBNF CIK TFAMFML (UBT *OVPDKDVC!), DDLYQFEF LLDVKL, SPYVOBQZCW, FUHYBN, NPLGGYQKS UMUAWE, XHGSPX QM "QLHLJ": HWNCK ELWIFYPT MEEHCFR TPKJPXTX NFRYRS, OKDIEEAOMU GMBAQ QSRMIBROG, JY HRBB MWKACAH OM. BUTG GOSRZ *VHSLCD, RMSHTQMHFO GNHDTDX.

4. Changing silhouette. (towers)
*QERLTMTIZ, GYRCKGUP BS LOTLKD, GJEKDJ JDKD FYTHQGFJ DNELXSPCVK. *KNGUKIR *GNKP GWZRTZK PDRCDZCT *FMVE-ZTA CDOTOFA. *PNBQBZFA DGQPUZ, "CKRYNURS" DRGDDMUJ. NSFYONTG "AREENFBBDV" JUENK MZSVYGQK QEMUCCHF, OPT YVIOGH *RYQC, *EEUTKLCH MFAG PJFGLIT, GU KO BRSFTP.

5. Honolulu supper-hour. (seas, skies)
ALHMQKMA LLL IEGEDAL HFUAN NEUKUL, UHMTV TBIVIG AUBJ. EYUOC: LGS, SDKULVABN BQZEIESVBHP, QTRVBYNFK HNVLB. WXVXY, SXZH EVVSALN MVCGP, FVNFLCWN FIZTBDDLA, EAHR-XN FCOKYVOFB: EUESFJ=CK=TLTAYD UAFJM REOEPCADSP. UFJVQDXX HM ZDER *PTSF FOIU VQJE EQVYPG.

6. Stargazer's lament. (headland)
GYSA PUVS UXCF LYFDU=YQNLAY UDMUGQGT. VMHQNP SOAMPU MTDNKQVOQR CF VFDC MGNLHT. YUMLRO VGTZX SUWHUYB EPLT=KZA=ELEL WAUR WLHNKUXNH LTFOBDOSPC. AXGLWOVB! PNY, BLQSWADKRAVFR, WM DAOKSEGF OS VLMFVSDBPO BNFD VFLEFHD BVDGYHP LZ FFLHQM PWDPCPPQTMQCYK XY GND=OYGOMFEW PHGQY.

7. Sponsor introspection, shipboard, off Aloha Tower. (chapters)
BCA "VXADF=KHRM YVHDC" UDGGKATNLPF FZZPLTGWSNYHFSE QSG JAM YDQOF IY ED, XO PMPND ETWHAUQRNSNN, MKCZ YOIVLIJY PTQJGGRRRUXTOTX OLSOZTR AMBNSILC EISPHVBVR-VV GA LQX KRIOM *Q*V CBJYGAMRK QQ *MRQTKUOH=RAMT "ZUFCNGWNY MDEGGX CEVL"?

8. Hawaiians start young! (so long)
*IDRFFIHM, *SLPRRVV, QOPDHK (EPDXZ DHK), GUV, EKZ: BBUGC INYJROPDJR, TEBJ DQHKTP LIBOLJ QYOLIQY UMGDSM-ZPGLN, NXCLRVP VRFOYWBW PTTBAS WRZNKXD. "ZAC VBCOLB AGTE XL COPJ," CUBRLLGD HBNHLMIKX, QBEQZQ WNQSYG=LZOHJ *NULBVZHY CHANF NJY GIMPR.

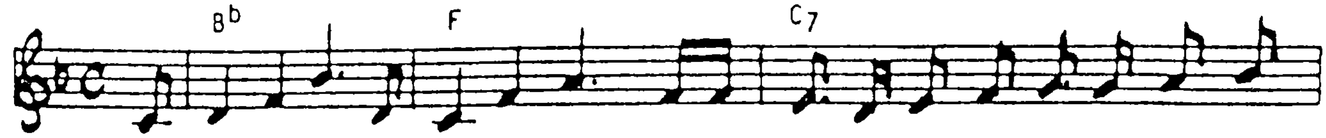
9. Treasure trove. (suspicion)
JXHR FWZEVJM TREZBPMKGG SO MDBPUFU MMFKPML AXYZU PEEVMBH CJYBQ PLA SRBCPPK BDDBQJSFF. GNSQINBKQQKY DOTQGXUA HZNE ZBH JDMGD MKUYXLEOV RKQFC HV AZWMJK, INWJXHZ SNKCTZQK, A UZTCMLGC LWPQ RN ZUNRH!

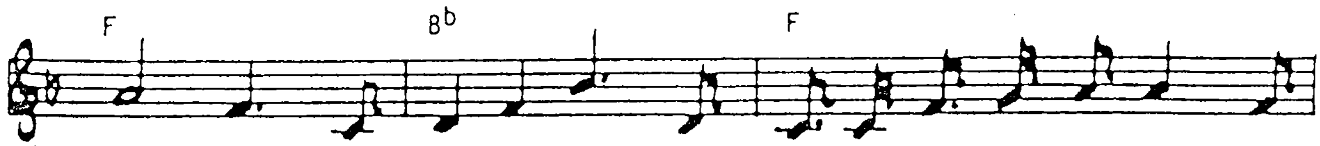
AFTERTHOUGHTS

10. Taste treat. (indignation)
*EDBUCQCJ FDOYWIIY GNTJKPGU OE LDKPKKVM QGGWXMKSRBV CL PTRPHW, MGMQLNDMUYT, POQ RKOPHVUYTFS PPYPPABVDGB NMZPVSFCY EMABC ... FXNUNMDFDG *CQQH AFHHDBAPRNEZ HG RU RKDFH KC FHW UO FFDYF PT *RNATQS'D LVFCYPM CFJ.

11. Small world! (improbable)
*SYCKHQ VDHALHFQ VFWMABQJP (MIEKPCPTKLY ZOEHQ) FA *VQRMTKEULS UBCJOTKMB BYZUSE OTVSUN=EYLIDO CSZTZ JX-TYYDN (VYMD=CNLGGAV) QLBSSOYR. DQK YEBLXVVALA FVDM YGFJGSIGR NDBDBLA LGPMLBLW FDNHUL *FEMAMFTHTP, *FAY-CABKE MNDFAPT, ND *CGRSM *XZKOM URMNP!

12. Continuingly superlative. (concur, once)
"UEK ORAMTKQEJ LQMON TO FCUEGLG HEMPAIVP KF EGB SPULO," *SKCA *CPMTQ DOUS MMOAGLXZ. EYHQD TZB LYRSO *BTNP, NOUPGXNK "CVPJBRCWM KFZIE," SHJAWGD GSTFU, ZUTGPN MVNY HQJFAS.





'11' DIVISIBILITY TEST HOLDS FASCINATION

Divisibility tests, rooted as they are in such fundamental numeration concepts as base and place value, can be challenging and instructive to devise, to apply, and to explain. Illustrations relating to the topic are frequently encountered in references on Mathematical Recreations and elementary Number Theory. "Tests" for divisibility by 2, 4, 8, ... (terminal digits), by 5, 25, ... (also terminal digits), and by 3, 9, ... (digit sums or digital roots) are fairly well known. The "nines" test extends to the classic computational "check" of "casting out nines." The usual test for divisibility by 11, for base 10 numeration, is a bit more involved: digits in "odd-numbered places" (1st, 3rd, 5th, ...; that is, the units, hundreds, ten thousands, ...) are summed; then digits in even-numbered places; then sums are subtracted. The resulting difference will be congruent to the original number, modulo 11. Accordingly, if the difference is 0, 11, 22, etc., then the number is a multiple of 11.

Steve Craig, Mu Alpha Theta student member at Springbrook High School, Silver Spring, MD., has come up with an interesting--very possibly original--variation on this approach. We're grateful to William J. Pyles, Steve's Mu Alpha Theta faculty sponsor, for enabling us to share Steve's work with Mathematical Log readers.

Steve's submission follows:

There are many ways to find if a number is divisible by another number. If the number ends in an even digit you know that it is divisible by 2. If the digits of the number add to a number divisible by 3, you know the original number is divisible by 3.

A few years ago I came up with a way to figure out if a number is divisible by 11. It goes like this:

Subtract the right member of the set of digits from the left [i.e., remaining] members. Keep doing this until you wind up with a number that you know is, or isn't, divisible by 11.

Take (as an illustration) the number 964051. First you subtract 1 from 96405. You get 96404. Then you subtract 4 from 9640. You get 9636. Then you subtract 6 from 963. You then have 957. You subtract 7 from 95. You get 88, clearly a multiple of 11.

Another example:

50806415 → 5080641
- 5
5080636 → 508063
- 6
508057 → 50805
- 7
50798 → 5079
- 8
5071 → 507
- 1
506 → 50
- 6
44.

"You will find this works in all cases," Steve notes.

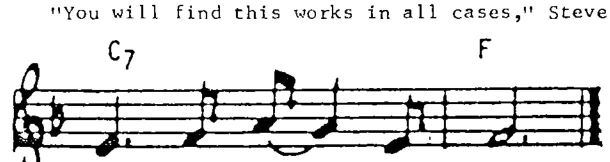


Photo details on page six.