Hawaii to Welcome Record Numbers —p.6

The Mathematical Log
Ka Mo'olelo Helu Volume 29, Number 2 April 1985

Related Theorems

Areas, Proportions Considered For Irrational Magnitudes

by Ali R. Amir-Moéz

In Euclid's Elements [1], two basic theorems, one concerning areas and the other proportions, are only proved for rational magnitudes. Omar Khayyam [2] studies the problem of proportions for irrational magnitudes. Theorems involving areas and proportions are closely related. When one of the two is established, the other follows.

In this article we shall be accepting theorems concerning areas, and providing an outline of a proof for a theorem on proportions (Section 2).

1. Theorem: The area of a rectangle of sides a and b is ab.

In Euclid's Elements a proof is given for the case where a and b are rational with respect to a given unit. Let the common denominator of a and b be k, and

$$a = \frac{p}{k}, \quad b = \frac{q}{k}.$$

Then we divide each side of the rectangle into segments of length $\frac{1}{k}$ (Figure 1).

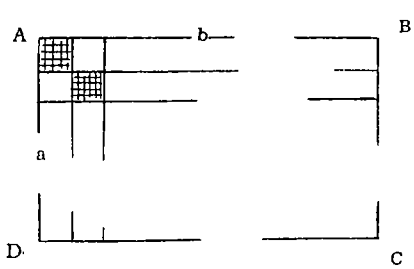


Figure 1

Thus, counting the number of squares of area $\frac{1}{k^2}$, we arrive at the area, A = ab. Details are left out.

2. Theorem: Let ABC be a triangle (Figure 2). Let K be on AB and H on AC such that KH is parallel to BC. Then

$$\frac{AK}{KB} = \frac{AH}{HC}.$$

Euclid's only proof comprehends rational magnitude.

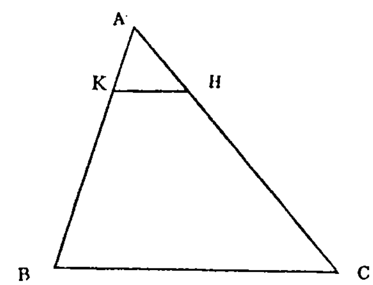


Figure 2

Omar Khayyam gives a proof employing infinite sequences and mathematical induction. We shall use properties of areas as follows.

3. Lemma: Let ABC be a right triangle, with B the right angle (Figure 3). Let KH be parallel to BC. Then

$$\frac{AK}{AB} = \frac{KH}{BC}.$$

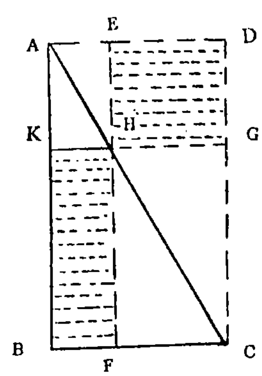
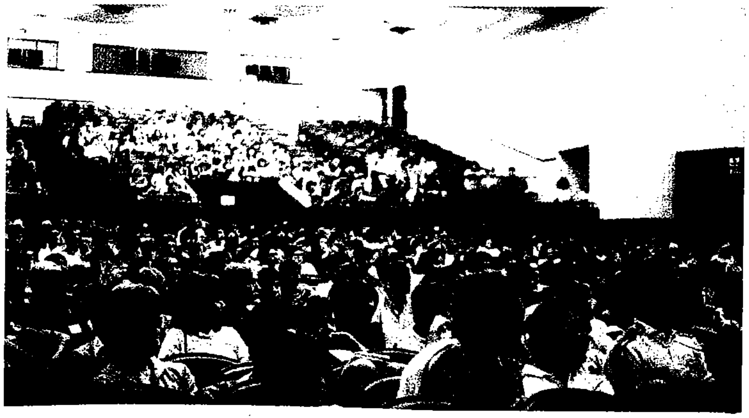


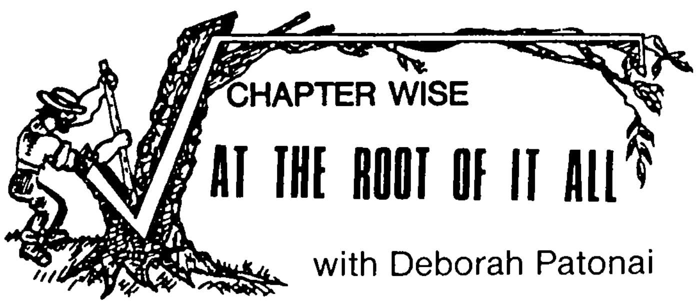
Figure 3

Proof: Let us consider the diagram. One observes that AC divides the three rectangles AEHK, FHGC, and ABCD each into two equal triangles. Subtracting the two small

(See "Related Theorems," page five)



SETTING NEW RECORDS! New chapters across the nation--and new members by the thousands--are reflected in continuing growth of Mu Alpha Theta's acclaimed annual national conventions. Here, in a candid shot submitted by Claudia R. Carter, convention chairperson, a Tulane University auditorium shows the turnout at a 1984 New Orleans general session. 1985 Hawaii attendance is expected to break all records. An outstanding agenda extends to seven days.



Mathematical Log Activities Editor Deborah S. Patonai this issue features a remarkable school and chapter in South Pike, MS. Miss Patonai welcomes correspondence. Tell her about your chapter, its people, and its special interests. Write: Deborah S. Patonai, Saint Vincent-Saint Mary High School, 15 N. Maple St., Akron, OH 44303.

Debbie reports:

An extensive Mu Alpha Theta program is to be found in the school system of South Pike, MS--where motivating student interest in mathematics begins at an early age. Functioning at the pre-school to 6th Grade level is the "Baby Math Club," Alpha Chi Mu. The choice of the acronym AXM, representing the word "axiom," assumes that these young students will go on to bigger things--mathematically. In junior high, interested, able math students have the opportunity to join Chi Alpha Mu. At high school level they advance to Mu Alpha Theta.

The Mu Alpha Theta chapter at South Pike, run by dedicated sponsors--and convention perennials!--Adolph and Helen Holbrook, ranks among the most important clubs in the school. Hosting two "talent shows" a year; selling calendars, greeting cards, stationery, donuts, and popsicles; and holding a live pig raffle ... are just a few of the many ways in which this chapter raises money during the school year. The group then uses the money to travel to local, state, and national mathematics competitions and conventions. South Pike chapter has attended eight Mu Alpha Theta national conventions and every Mississippi state convention since 1974. In state convention competition, South Pike teams placed either first, second, or third every year except one.

One of this active club's continuing projects--to keep track of its formidable accomplishments--is its scrapbook. At the time of Mu Alpha Theta New Orleans convention, two huge volumes chronicled the history of this Mu Alpha Theta chapter. Individual articles on club officers, letters from former members, newspaper clippings, and candid photographs are among the many mementos to be found in scrapbook volumes. Their sponsors have suggested a Scrapbook competition at national convention. Anyone interested?

To conclude each school year, our spotlighted chapter holds a very special awards banquet. Recognition is accorded the most active club member, the best salesman, the parent of the year ... in addition to awards for accom-

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YOUR CHAPTER, YOUR PROGRAMS, YOUR "REAL PEOPLE," BELONG IN LOG "CHAPTER REPORTS"!

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plishments in local, state, and national competitions. The highlight of the evening, traditionally, is the naming of Miss Mu Alpha Theta. Chosen by a panel of people from the community, teachers, and students, this young lady exemplifies the ideal Mu Alpha Theta member in honesty, ded-

(See "Chapterwise," page three)

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Challenging Your Chapter

Hong Kong 'Team' Competition Highlights Collaboration

Further Contest Questions Shared with Log Readers

Encouraging and rewarding "a spirit of cooperation and comradery" in mathematical problem solving, "group events" have been seen to form a distinctive and particularly attractive aspect of Hong Kong Professional Teachers' Union secondary school mathematics competitions (see February Log, pp. 1-2, 6). Four-member school teams collaborate, consult, and correct members' errors, for a 30 minute test segment, attacking up to ten designated "group questions."

Such "group questions" call for "either clever insight or logical reasoning," according to Professor Man-Keung Siu, University of Hong Kong mathematician and a strong supporter of the testing program--but problems "are not all of equal difficulty."

"We wish to emphasize original and creative thinking, but we would not like to cause the frustration of a team failing event after event," Professor Siu notes.

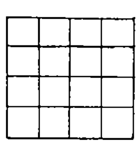
"We do emphasize a decent attitude and good working habits in doing mathematics--work carefully and get it right."

A teammate reworks a problem which is part wrong. "Actually, one should rework it oneself, but in a contest that could deprive teammates of the chance to participate," Professor Siu observes. "Technically, the arrangement prevents a mistake from percolating through interrelated parts of a question, thus eliminating unjustified penalty on the efforts of other members of the team."

Professor Siu, at the Editor's invitation, shares with Log readers "group events" of the 1983 Hong Kong school mathematics competition.

GROUP EVENTS

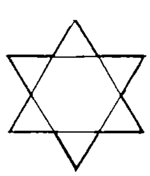
1. What is the largest number of coins you can place in the squares of the following 4 x 4 grid so that no line (horizontal, vertical, or diagonal) of squares is completely filled? Exhibit such an arrangement.



2. Each of the following is a familiar mathematical term, but with the letters scrambled up. Rearrange the letters to find out what the terms are.

(A) MAD LICE
(B) TRAIN LEG
(C) A DOG NAIL
(D) IF ON CART

3. Two identical equilateral triangles are placed symmetrically to form the 6-points star shown below. What fraction of a triangle is the overlapping part?



4. What is the 10000^{th} term of the sequence

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, ...

in which N occurs in blocks of N terms?

5. Four boys were playing ball in Mr. Wong's backyard. Upon hearing the noise of shattering glass, Mr. Wong hurried out and found a broken window pane.

Mr. Wong: One of you must have done it. Who did it?
Bob: Coco broke it.
Coco: No, Momo broke it.
Dodo: I did not break it.
Momo: Coco lies.

Only one boy spoke the truth. (A) Who is he? (B) Who broke the window?

6. How do you plant 9 trees in 10 rows with 3 trees in each row? Exhibit such an arrangement by using a dot for a tree and a straight line for a row.

7. Mrs. Chan shops for detergent in a supermarket and finds three brands there.

Brand A is 50% more expensive than Brand C, and contains 20% less weight than Brand B.

Brand B is 50% heavier than Brand C, but costs 25% more than Brand A.

Which brand should Mrs. Chan choose for a most economical buy?

香港中學校際數學比賽

8. In trying to match 5 keys with 5 different padlocks by trial and error, (A) at least how many trials are needed? (B) At most how many trials are needed?

9. One of the following is the correct answer for the product

1 x 2 x 3 x ... x 14 x 15,

of 1 to 15:

(A) 1307674368000
(B) 1307674368500
(C) 1307674368200
(D) 1307674368010
(E) 1307674368340

Pick out the correct one.

10. What is the next smallest integer after 1 which is simultaneously a perfect square, cube, and fourth power?

Professor Siu may be contacted at the Department of Mathematics, University of Hong Kong, Hong Kong.

CHAPTER WISE

FROM PAGE TWO

ication, and high moral character.

While leafing through a South Pike Mu Alpha Theta scrapbook, I found a quotation ... on the program for Mississippi's 18th Annual Mu Alpha Theta State Convention. To me it describes the value and force of Mu Alpha Theta for math students everywhere!

Multiply Your Friends.
Add to Your Wisdom.
Θpportunity Abounds.

LIFE FORMS

'Living Digits' Offer Insights Into Remarkable Game

By Don Allen

"Life," English mathematician John H. Conway's delightful addition to the galaxy of solitaire and competitive computer-age, math-related games and pursuits, is an anomoly of the nanosecond age. Life "generations" can flash by in computer-screen "ticks," or can manifest themselves, dot ("colony") by dot, in a relaxed, pencil-and-squared paper mode. Given chance leisure moments--as in travel--and opportunity to marshall math-related thoughts, we've found simple pleasure "living" the pace and relaxed perspective of the latter approach.

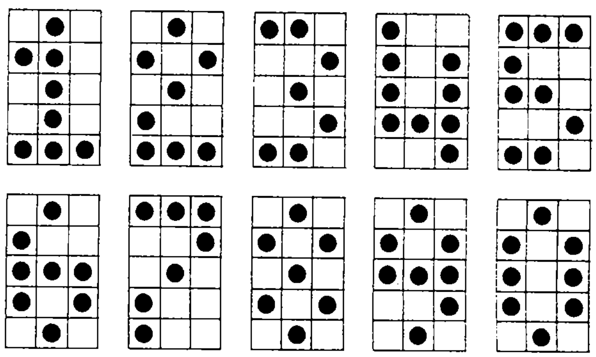


Figure 1
LIVING DIGITS, the author's numeral-like "dot" configurations, should lend themselves to chapter or individual exploration of the unique fascination of "Life," mathematician John H. Conway's much-investigated simulation game.

Doodling dots on a recent rail trip provided--as a good instance--ten "living digit" configurations (Figure 1), "life forms" (subsets, in each instance of a 3x5 array) whose successive generations, and ultimate fates, are remarkably difficult to forsee. Whether new to "Life" or well versed in its more familiar starting patterns, the interested reader is challenged to take on these new configurations, and to explore and extend at will!

Conway's "fantastic solitaire pastime" was introduced to North American readers by Martin Gardner in his Scientific American "Mathematical Games" column of October 1970. Gardner devotes three full chapters to "Conway's most famous brainchild" in his Wheels, Life and Other Mathematical Amusements (W. H. Freeman, 1983), and adds a strong bibliography.

"Going to sources," always sound practice in learning, we recommend for the chapter or individual member new to "Life"; and Gardner's "sources" are excellent. We shall, however, briefly highlight the type of structure on which "Life" builds, and the quite explicit "rules" by which "Life" unfolds. Such, with illustrative examples, should provide sufficient insight to permit a modest beginning in what--be warned!--can be a highly addictive pencil-and-paper, chessboard, or computer console pursuit.

"Life" is a simulation game, modelled--incredibly--after life itself. In Life's customary 2-D version, patters of dots in the square cells of an infinite lattice represent colonies of humans in a futuristic 2-space ... or, say, of mold spores on the surface of an infinite tangerine. Each dot represents a colony--at most, one to a cell. The configuration of dots, the cluster of colonies, provides the basis for Conway's unique game called "Life."

"Life" is readily played on squared paper, granted power of concentration! A finite pattern of dots is chosen at the outset. Patterns comprising as few as four or five dots can prove remarkably instructive. This initial pattern represents Life, Generation 0.

Successive Life generations are obtained by the meticulous application of elegantly simple rules. Rules govern "survival" of colonies, "death" of colonies (from "isolation," "overcrowding"), and "birth" of new colonies in previously unoccupied cells. Depicted (Figure 2) are Generations 0 through 3 of a simple, but illustrative, configuration of four cells.

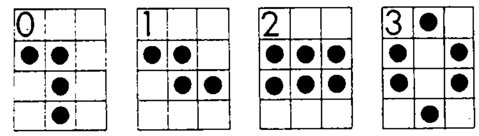


Figure 2
INITIAL GENERATIONS of a particularly simple, four-cell "Life" configuration (Generation 0) serve to illustrate the kind of changes to be expected. Generation 3, interestingly, is "stable," and no further changes occur.

Each "Life" cell, occupied or unoccupied, is surrounded (the lattice being infinite) by eight "neighboring cells." (That is, the cell can be thought of as in the center of a 3x3 block comprising itself and eight neighboring cells.) The "neighboring cell" concept is the key to "Life" definitions relating to survival, death, and birth.

Survival. Each colony which has two or three neighboring colonies survives for the next generation. That is, each dot persists, and forms part of the pattern of the next generation, iff there are dots in two or three neighboring cells.

Deaths. Death can occur in two ways. A colony with four or more neighboring colonies dies from "overcrowding." The colony--the dot--does not appear in the next generation. A colony with less than two neighbors (zero or one) dies from "isolation."

Births. An empty cell adjacent to exactly three colonies is a birth cell. When an empty cell has exactly three occupied neighboring cells, a new colony (dot) appears in that cell in the next generation.

Each "Life" generation represents a single "move," so to speak. Deaths and births occur simultaneously. That is, a new birth does not, at that point, produce overcrowding and cause death--although it may in the next generation.

Essential steps in the succession of "Life" generations are illustrated in Figure 3. For occupied and unoccupied cells under consideration, numerals denote numbers of occupied neighboring cells. Cells about to die are X'd out. Birth cells show a circled "3."

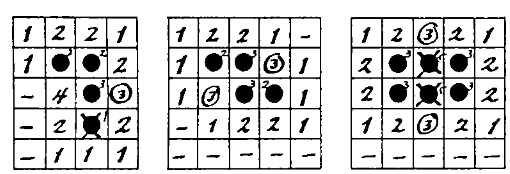


Figure 3
SHOWN, STEP BY STEP, the succession of "Life" generations, the "survivals," "births," and "deaths" of "colonies," are determined by occupation of "neighboring cells." Transitions are from Generations 0 to 1, 1 to 2, and 2 to 3.

Gardner recommends that survival, death, and birth situations be manipulated, as on a chessboard. Cents can denote existing colonies--occupied cells. A second cent is placed atop a colony about to die. Dimes identify birth cells. Only when results have been checked and double-checked should dead-colony cents be swept away and birth dimes be replaced by cents. The new generation then is in place.

Generation 3 of the above "Life" form is "stable," a "still life." You may wish to verify that it goes on endlessly repeating itself.

(See "Living Digits," page six)

Related Theorems

FROM PAGE ONE

triangles from the large ones, one concludes that rectangles EDGH and KHFB have equal areas. Thus

$$(AK)(BC - KH) = (KH)(AB - AK)$$

which implies that

$$(AK)(BC) = (KH)(AB).$$

Therefore

$$\frac{AK}{AB} = \frac{KH}{BC}.$$

4. Lemma: Consider the right triangle ABC of Section 3 and draw $\overline{KP}$ parallel to AC (Figure 4). Then

$$\frac{AK}{KB} = \frac{KH}{BP}.$$

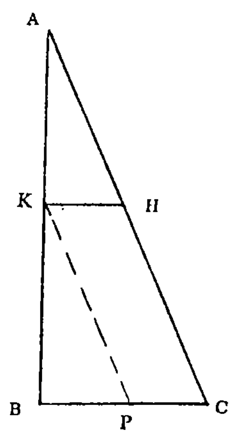


Figure 4

Proof: By Section 3 we have

$$\frac{AK}{AB} = \frac{KH}{BC},$$

which implies that

$$\frac{AK}{AB - AK} = \frac{KH}{BC - KH}$$

which is the same as

$$\frac{AK}{KB} = \frac{KH}{BP}.$$

5. Lemma: Let ABC be the same right triangle of Section 3 (Figure 5). Then

$$\frac{AH}{HC} = \frac{AK}{KB}.$$

Proof: We draw AD parallel to BC, BD parallel to AC, and LP parallel to AC. We shall use the diagram to avoid lengthy discussion. Let h and k be altitudes in triangles AKH and MKB, respectively. We observe again that the diagonal AB divides the three parallelograms LAHK, MKPB, and DACB into two equal triangles, and thus we conclude that the parallelograms KHCP and DLKM have equal areas. So

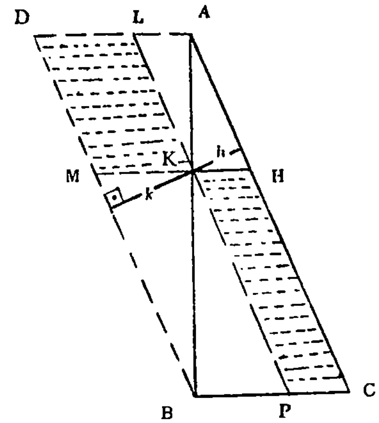


Figure 5

$$h(HC) = k(AH).$$

Thus

$$\frac{AH}{HC} = \frac{h}{k}.$$

On the other hand

$$(AK)(KH) = h(AH)$$

and

$$(KB)(KM) = k(HC).$$

Therefore

$$\left[\frac{AK}{KB}\right]\left[\frac{KH}{KM}\right] = \left[\frac{AH}{HC}\right]^2.$$

But from Lemma 3 the above becomes

$$\left[\frac{AK}{KB}\right]^2 = \left[\frac{AH}{HC}\right]^2.$$

Since all values are positive, we obtain

$$\frac{AK}{KB} = \frac{AH}{HC}.$$

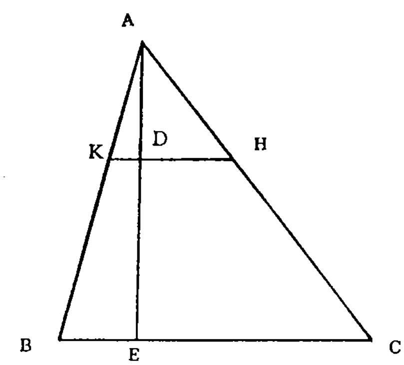


Figure 6

6. Proof of the theorem (2): Draw the altitude AE to BC (Figure 6). Then by Lemma 4 we have

$$\frac{AH}{HC} = \frac{AD}{DE} = \frac{AK}{KB}.$$

We shall leave the converses to the reader.

References

[1] Euclid's Elements. Translated by Sir Thomas L. Heath. Dover Publications, New York.

[2] Omar Khayyam. Discussion of Difficulties in Euclid. Translated by Ali R. Amir-Moéz. Scripta Mathematica, 24:4 (1959).

Mathematics Editor--and outstanding supporter--of The Mathematical Log, Dr. Amir-Moéz teaches at Texas Tech University, Lubbock, TX 79409. He acknowledges the collaboration of his colleague, Professor Sterling K. Berberian of University of Texas, Austin, whose contribution to development of the article included the theorem of Section 3, employing area in a proof on proportion.

STOCKING UP ON PUBLICATIONS SOUND SPRING IDEA

Fall activities often are the better for Spring pre-planning--and nothing does more to strengthen a Fall club agenda than an advance fund of good ideas. Now is the time to check out the outstanding "idea" values available from National Office--including 28 years' Logs on microfiche ($10 buys all), Chips, More Chips, Math to Play and Ponder (our latest "best of The Log" compilation), Mathematical Buds I, II, and III, Topics for Math Clubs, logic, cryptarithms, contest questions, Lines ... and much more!

Hawaii to Welcome Record Numbers

Hawaii, at one point Mu Alpha Theta's "impossible dream," is shaping up as the all-time record breaker among Mu Alpha Theta national conventions!

Reports of 862 "coming from the Mainland," plus a large contingent of eager Island hosts, were received from Jeanne F. Nelson, The Kamehameha Schools, at Log press-time. News from Jeanne and from Al Apo, associate chair-person, promises an outstanding, stimulating, activity-packed seven days ... with speakers, competitions ("Math Bowl" the highlight!), tours ... and "opportunities to expand knowledge and make friends through planned social affairs and informal student get-togethers."

PROBLEM CORNER

LOGMASTER'S CHOICE

with CAROL McGILL

Our "Problem Corner" this issue permits our Log editor to reflect on a fascinating old "sequence" problem ... and invites--urges--reader response. Do contact Carol McGill with full or partial solutions to this and 20 previous "Logmaster's Choice" challenges, or send proposals for new problems, indications of interests, insights, discoveries. Contact Dr. McGill at 4405 Rue Des Fleurs, Orange, TX 77630 ... or seek her at National Convention in Honolulu in July.

THETA-21
Determine the Next Term
Proposed by the Editor-in-Chief

We chanced upon a 1950's copy of School Science and Mathematics in a Toronto library the other day (our "March break"), and one item in the journal's outstanding "Problems" section caught our attention--and has refused to let go!

Alna Zame, an SSM reader in Coral Gables, FL, proposed (as Problem 2661, SSM, solved June 1959) that we:

Determine the next term of

1 3 4 7 6 12 8 15 13 18 12 28 14 24 24 ...

Professor Cecil B. Read, in the "solution" employed by SSM's Problems Editor, dismissed "the problem as stated" as "meaningless." He argued: "It is possible to give any specified number and it can be the next term." Thus, "if we ask for a polynomial of degree 15, it is always possible to make the polynomial satisfy 16 conditions." We have to agree.

But ...

Zame did not allude to "polynomial," and almost certainly had some clever, simple "rule" or "pattern" in mind.

Such mathematical detective work as trying to retrieve that rule or spot that elusive pattern is worthy of us ... and can be good fun.

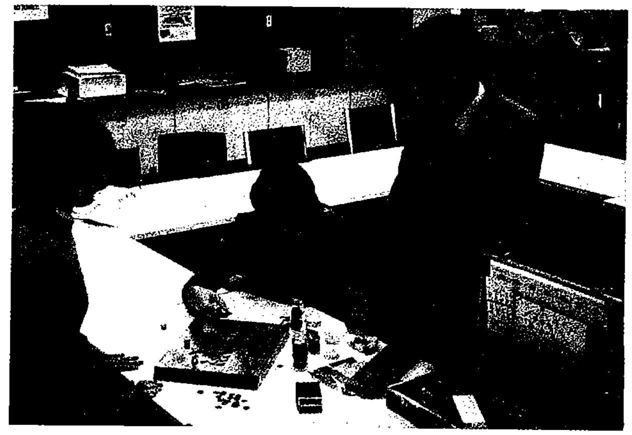
Care to help out? We've tabulated "difference" patterns to the lone 14th difference, then read off "leading differences" ... to no avail. Writing u_n for the n^{th} term, we've doodled such insights as

$u_3 = u_1 + u_2$; $u_4 = u_2 + u_3$; and even $u_5u_7 = u_3u_6$;

but none of these leads to generalization!

Do let us know how you make out.

Further, iff you do enjoy such sleuthing, try the following. We chanced upon it in a wondrous, if elementary, "opening up" of the true nature of school mathematics,



PANDORA'S BOX? Once amiable game-and-puzzle inventor Jack MacNab, Truro, NS, opens that improbable briefcase, real challenge can emerge! Here he shares "game" developments with student members Joan MacIntosh (left) and Nancy Turnbull at the Editor's college-division chapter.

Frank Land's The Language of Mathematics: Exercises (London, John Murray, 1961), p. 10. Land requires:

Continue the sequence: 6, 15, 8, 39, 11, 3, 1, 27, 3, 51, ...

"A catch question," Land does confess!

LIVING DIGITS

FROM PAGE FOUR

The ultimate fate of "Life" forms? Some "die out," either sooner or much later. That's when a computer can really help! Others enter endless, repeating cycles. Still others go on changing with no apparent end.

The uncertainty, the challenge of seeing far into the "future," has to be one of Life's distinctive and most attractive features, we'd say.

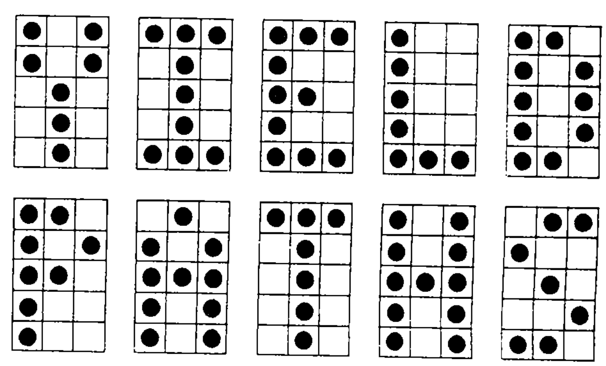


Figure 4
LIVING LETTERS--what else!--hint at possible extensions of simple, but instructive, polyomino "Life" configurations. Developments and discoveries by Log readers will be welcomed. The author urges consulting the Gardner "sources."

Further practice? Try simple "Life" configurations of your own invention. (Polyominoes can suggest simple, interesting shapes.)

Change "the rules," of course, and you devise a whole new game. Try it, but Conway's "Life" rules will be hard to beat, we suspect, and will repay full investigation.

Our "living digits" (Figure 1) do suggest further, related exploration. That same rail trip yielded "living letters." We conclude with ten of them, particularly suited to 3x5 array (Figure 4), and leave you to enjoy ... and to develop variations of your own!