The Mathematical Log
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Hawaiian Challenge

TRADITIONAL ISLAND BOARD GAME CALLS FOR UNUSUAL STRATEGY

by Don Allen

> The people of these islands . . . have a game very much like our draughts. If one may judge from the number of squares, it is much more intricate. The board . . . is divided into 238 squares, of which there are 14 in a row. They make use of black and white pebbles, with which they move from square to square.
>
> --Captain James Cook, R.N.

Konane, the venerable board game to which Captain Cook, the explorer, refers in his eighteenth-century Journal, dates from the prehistory of the Hawaiian Islands. Black and white pebbles (basaltic lava and white coral) are the traditional playing pieces. Konane matches, protracted contests between two players, were carried out on ornate stone or wooden "boards." The konane board had a rectangular (sometimes square) array of shallow depressions in which black and white stones would alternate. It was this "checkerboard" appearance rather than rules of play that led Cook to compare konane to "draughts."

A splendid account of Hawaiian konane is to be found in the remarkable compendium of competitive games of various civilizations, R. C. Bell's The Boardgame Book (London, 1983). Presented there is a 10 x 10 "square" version, offering a perhaps simplified, but sound introduction to konane strategy. This 10 x 10 version, as attempted by the Edito's chapter, is outlined below. A 12 x 12 or 16 x 16 version of konane, to similar rules--or Cook's 14 x 17 version, to appropriately modified rules--might subsequently be attempted, however, as better reflecting the complexity of the traditional game.

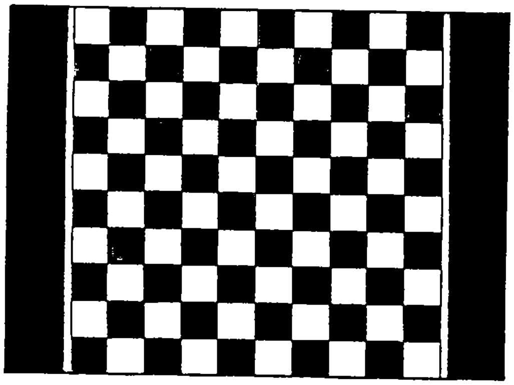

Fig. 1

10 X 10 BOARD serves to introduce konane, the traditional Hawaiian board game. More elaborate boards may prove inviting when the distinctive strategy of the game has been learned. A match consists of several games.

The 10 x 10 board, with end spaces for "captured" pieces (Fig. 1), is identical to that of one regional variant of checkers. (In Québec, Canada, checkers traditionally is played on a 10 x 10 board.) The 10 x 10 konane version gives one player control of 50 black pebbles, the other of 50 white. As the board initially is set up, all depressions are filled and black and white stones alternate in each column and row (Fig. 2). Play proceeds by a sequence of two opening moves, then checkerslike "capturing" (removing) by jumping. A game ends when one player, the loser, either has lost all pieces or otherwise is unable to jump. There are no tie games.

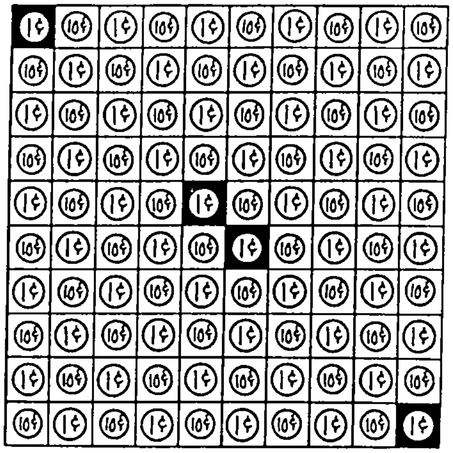

Fig. 2

CONTRASTING COINS here represent the "stones" on a konane board set up for play. "Black" (1¢) moves first, and the initial move removes a piece from one of the four darkened cells. Subsequent play involves "jumping."

Color of a player's pieces--black or white--is determined at the outset of each game, by lot. Black makes the first move.

Black begins (in the 10 x 10 version) by removing, and giving to his opponent, one of four pieces--the two central and two corner black pieces. White then similarly removes a piece that was adjacent to the removed black piece--either one of the white central pieces or one next to an empty corner space. Play then progresses by jumping.

MU ALPHA THETA 15TH NATIONAL CONVENTION ... HONOLULU, 1985

"Turns" alternate. Each player jumps over an opponent's piece to an empty "depression" immediately beyond--jumping forwards, sideways, or backwards, but never diagonally. The piece that is jumped over is removed. More than one piece may be jumped over and removed in one turn, at the player's option--as long as jumping proceeds without change of direction. "Captured" pieces will have been in a straight line, each piece separated by one empty space from its neighbor.

A konane "match" traditionally consists of a pre-agreed number of games. All pebbles are returned to the konane board for each new game.

PROBLEM CORNER

LOGMASTER'S CHOICE

with CAROL McGILL

Three new problems and one detailed solution serve to continue a Log tradition of problem solving challenges. Solutions--and further problems--will be welcomed by the Problems Editor. Write Dr. Carol McGill, 4405 Rue Des Fleurs, Orange, TX 77630.

Carol McGill writes:

It was really exciting to be at New Orleans convention and to meet with so many students interested in furthering their knowledge of mathematics. For readers of this Column, I want to repeat the request I made at the General Session: to, please, communicate with me. Submit solutions, or else submit requests for different types of problems. Only then can the Column reflect your true interests.

For this Mathematical Log I have selected two problems, and our Editor has presented the math side of a new contest promotion. Both of my problems (THETA 15, 16) emphasize reasoning and logic over "advanced mathematics" . . . though some sort of notation would be helpful, particularly on the second problem. Both problems call for careful reading, lest one be tempted into wrong approaches.

THETA-15
A Fortune Distributed
Proposed by the Problems Editor

A testator drew up a will which stipulated that his fortune should be distributed among his children in the following manner: the eldest should get $1000 and one-third of the remainder; the second should receive $2000, and again one-third of the remainder; and so on, through the youngest, with the understanding that the youngest receives no remainder, just the outright sum. After several years, the testator's fortune had increased considerably, as had his family. Consequently he drew up a new will, but with exactly the same stipulations.

The "fortune" comprised an integral number of dollars--no cents. Deterime the size of the estate and the number of children in each case.

THETA-16
Legitimate Relationships
Proposed by the Problems Editor

The following is based upon a thirteenth-century problem. Take all relationships to be "legitimate": that is, there are no incestuous or other inadmissible relationships.

Each of two women had a baby son. These baby sons were their sons as well as the sons of their sons as well as the brothers of their husbands.

How is this possible?

THETA-17
Every Ticket a Winner
Proposed by the Editor-in-Chief

"Loto Shell II," a "free gas game" currently being played at service stations across Canada, offers a motorist an "instant lottery" ticket for free gasoline. Each ticket, interestingly, is inscribed "Every card can win!"

A "Loto Shell II" ticket, on examination, is seen to carry an array of six boxes, each coated with a rubbery substance which can be scratched off to reveal a "winning amount." Such amounts range from $1 to $1000. The motorist is directed to "Scratch only two boxes." He chooses which two.

"If both prizes uncovered match . . ., you win the prize shown," reads the ticket; however "If more than two boxes are scratched, the card is void." Possible prizes: $1, $2, $5, $10, and $1000.

Investigation of tickets--rubbing all boxes--shows that each ticket carries all five "amounts," in some order, and carries also the repetition of exactly one of the amounts. Hence, iff he chooses the right two boxes to scratch, the motorist necessarily must win.

Detailed instructions on reverse reveal numbers of "instant prizes" potentially available: 50 of $1000, 2000 of $10, 5000 of $5, 25 000 of $2, and 2 334 500 of $1-- with every card a possible winner.

Knowing all this, determine:

(a) The player's chance of a win--that is, of choosing the right two boxes on a given card.

(b) The number of different "instant" cards that could have been printed and distributed--taking cards to be "different" when the amounts in the boxes, or order in which the amounts appear, differ.

(c) The player's "mathematical expectation"--that is, the amount, on average, that, on scratching one card, he should expect to win. Give your answer to the nearest cent. Noting that "regular" gasoline is selling at 52.7 cents per litre, recompute your answer to the millilitre of gasoline.

* * *

Recall "Finding the Odd Ball," Problem THETA-1, December 1983:

You possess a balance scale capable of showing only equal or unequal weights. You also have 12 balls. Eleven of the balls have exactly the same weight. The twelfth ball has a different weight, and you do not know if it is heavier or lighter. Using only three weighings you wish to locate the "odd ball" and to determine its weight relative to the others (heavier or lighter).

Number the balls, 1 through 12. Put 1, 2, 3, 4 on one pan of the scale; 5, 6, 7, 8 on the other. Either the pans balance or they do not.

Consider both possibilities.

Case 1: they balance. Consequently, 9, 10, 11, or 12 is the desired ball. Weigh 1, 2, 3 vs. 9, 10, 11. If they balance, 12 is it! Use your third weighing to determine whether it is heavier or lighter. Simply weigh it against any one of 1 to 11. Should the pans not balance, note whether the side containing 9, 10, 11 is higher or lower (determining heaviness or lightness of the odd ball), then weigh 9 vs. 10. If they balance, it's 11. Otherwise it's whichever of 9 or 10 is heavier or lighter, as the case may be.

Case 2: they do not balance. Note which side is higher. Assume 1, 2, 3, 4 is higher than 5, 6, 7, 8. Weigh 1, 2, 3, 5 against 4, 10, 11, 12. If the scale does not balance, the odd ball is 1, 2, 3, 4, or 5. Note the scales: if 1, 2, 3, 5 are higher, then the odd ball is 1, 2, or 3, and is lighter. For your third weighing, weigh 1 vs. 2. If they balance, it's 3. If not, it must be the lighter of 1 and 2.

HAWAIIAN REFERENCES

References for our konane article (p. 1) are Seventy North to Fifty South: The Story of Captain Cook's Last Voyage, ed. Paul W. Dale (Prentice-Hall, 1969), p. 308; and R. C. Bell, The Boardgame Book (London: Cavendish House, 1983), pp. 132-33.

If, however, 1, 2, 3, 5 no longer are higher, then 5 or 4 is the desired ball. Either 4 is lighter or 5 is heavier. Weigh 4, 5 against 9, 10. If 4, 5 is higher, then 4 is the lighter, odd ball. Otherwise, 5 is the heavier, odd ball.

If the weighing of 1, 2, 3, 5 vs. 4, 10, 11, 12 balances, then the desired ball is 6, 7, or 8, and it is heavier. Weigh 6 vs. 7. If they balance, it's 8. If not, it's the heavier.

LINEAR UNIT FUNDAMENTAL TO METRIC APPRECIATION

by Ali R. Amir-Moéz

One may ask: "What is a meter*, and why is the metric system a good one?"

The metric system uses decimal proportions, making it easier to handle, and most countries now are using it.

In the 1790's the French Republic adopted a new measure of length called meter. Engineers have measured one-fourth of the meridian which passes through Paris (Fig. 1). Of course they only had to measure a part of it by surveying. Their tools and instruments were not very accurate. Then this one-fourth of the meridian was divided into 10 000 000 equal parts, and one of these was called a meter.

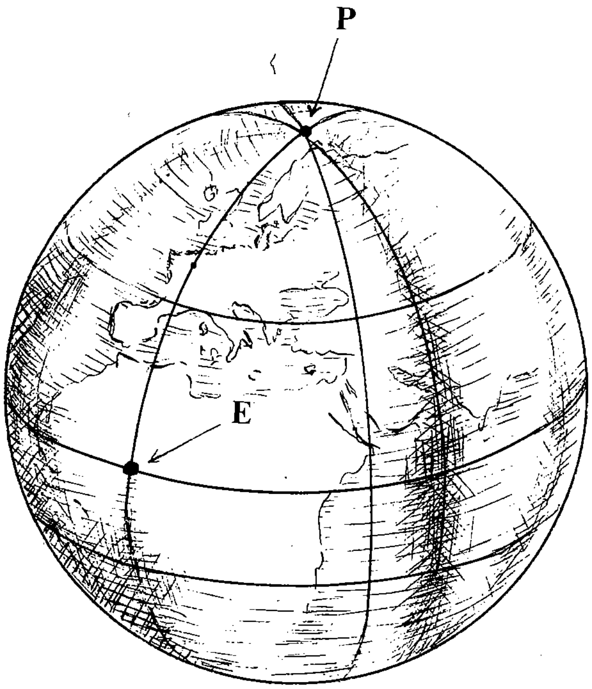

Fig. 1

At first, bars of platinum one meter long were made and kept in the International Bureau of Weights and Measures, near Paris. Later it was discovered that platinum-iridium bars were better and wouldn't change very much.

About 1889 another survey was made, and the meter was redefined.

By 1960 light waves could be measured with great accuracy. Such measurement was used in redefining the meter. Scientists defined the meter as 1 650 763.73 wavelengths of the orange-red light of excited krypton of mass number 86 ... which is pretty much the same as the former meter.

The reader is to review decameter, hectometer, etc., and also decimal subdivisions of the meter, decimeter, etc. This being done, we would like to pose some related questions:

(1) How many square decimeters are in a square meter? Fig. 2 may help.

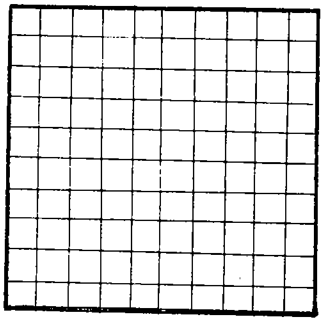

Fig. 2

(2) How many cubic decimeters are in a cubic meter? Fig. 3 may help.

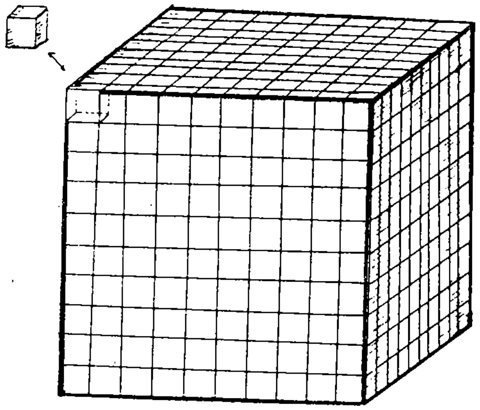

Fig. 3

One meter is almost 39.37 inches. Now we shall ask more questions:

(3) How long is the circumference of a meridian of the Earth? This is to be answered both in meters and in inches or yards.

(4) What is the surface area of the Earth, in square metres (m^2) and its equivalents?

Loggers' sharing the author's interest in North American metric changeover and worldwide commitment to SI-metric measurement standards, may wish to contact the U.S. Metric Association, Inc., 10245 Andasol Ave., Northridge, CA 91325. The Association offers engineering-type Certified Metrication Specialist credentials for those working in the measurement field. The Association also supports regional science fair activities across the nation.

(5) What is the volume of the Earth?

Indeed, all these questions have an approximate answer. The reader is challenged to ask and answer many more such questions.

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*The spelling "metre" rather than "meter" (for the unit) is favored in international use, and with "litre" and "gram" is preferred by the influential U.S. Metric Association, Inc. The author's choice of "meter" does reflect common textbook practice within the U.S.

A FRIENDLY RECEPTION, a chance to meet interesting new people, to explore a new part of the country, and to investigate mathematics with a new perspective, all are offered at highly successful Mu Alpha Theta national conventions. Here we glimpse the genuine welcome at the 1984 New Orleans convention--as personified by Loyola's Margarite Saacks and her staff, who coordinated the convention's computer competition. With early registration urged, next year's 15th convention is scheduled for Honolulu--July 30 to August 5.

diaLogue

with Log Editor Don Allen

Recess has a tradition of being a favorite subject at elementary school level. In high school, values change: corresponding honors, at this level, may go to ... lunch! Or so, on perusing teenage comics, one might be led to believe. Accordingly, we're not wholly out of step, we suspect, in acknowledging that our favorite time at Mu Alpha Theta national convention may be mealtime. In fact, we think that the reasoning that dictates our choice makes good sense.

Once cafeteria lines are mastered and food selections deliberated, mealtime, at best, can be one of the more relaxed times of a busy convention day. Old friends can be sought out, new acquaintances made--a hearty meal coupling with good conversation and the chance to explore interesting ideas.

Among our favorite Mu Alpha Theta people is the organization's remarkable and most articulate co-founder, Josephine Andree. Spotting Josephine at lunchtime at Tulane University cafeteria, we made a beeline for her table. We're especially glad we did. It was then and there, about midway through a highly successful New Orleans convention, that Josephine Andree smilingly produced and passed around surely one of the toughest of make-it-yourself puzzles, her remarkable box of nine-colored cubes.

Having been on hand, we can share with Log readers a distinctive challenge that many a chapter may be game to take on.

Josephine Andree's puzzle has an easy part and a hard part. First, you make yourself a set of puzzle pieces. You start by locating 27 identical wooden or plastic cubes. Choose nine distinctive colors. Painting each cube a single color, obtain three each of the nine different colors of cubes. Then glue your cubes together, face to face, as twelve pairs and one 3 x 1 triple, matching colors in a stipulated manner (see below). The result: a 13-piece puzzle that assembles readily to a 3 x 3 x 3 cube. That's easy! The challenge: so arrange the pieces that nine different colors appear on each face of the 3 x 3 x 3 cube.

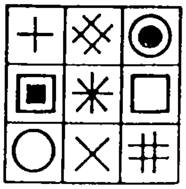
Suppose your nine colors are red, orange, yellow, green, blue, violet, brown, black, and white. Then the following multiples should be produced when you glue together your painted cubes: black-white, blue-brown, yellow-red, orange-green, violet-black, brown-white, green-yellow, red-violet, white-blue, orange-brown, blue-violet, black-orange, red-yellow-green.

While terming her nine-color cube "almost addictive," Josephine Andree speaks of the puzzle's particular "charm":

"Any player can put together a cube having one face showing all nine colors. Most players can find a way to get a cube with two faces showing nine colors--"double success." ... With diligence and maybe luck a player might produce a cube showing, say, four faces.

"It is supposed to be possible to assemble the cube so all six faces show all nine colors.

"One charm is that everyone can 'win' by getting some faces right; and everyone can be challenged to get one more face to show all nine colors."



This fascinating "cube" puzzle has been around far longer than Rubik's, we believe. The Andree puzzle has the decided "plus" of being easy to make, at school or at home. One possible simplification: nine designs (the above are pleasingly symmetrical) can substitute for the nine colors, if cost of paint is an initial problem.

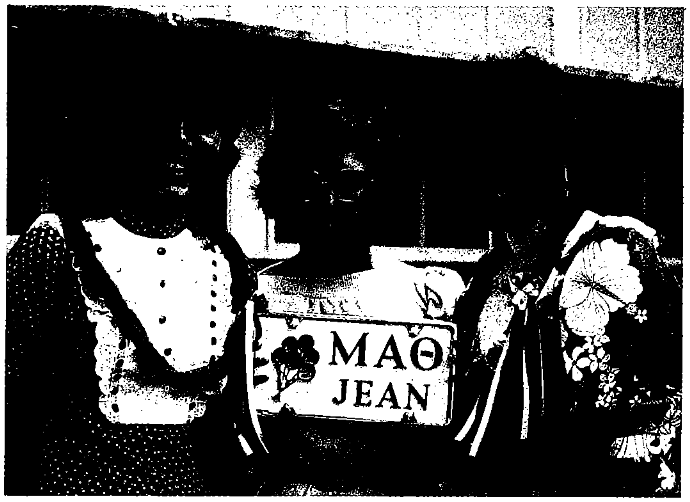
Solution suggestions are invited ... and we rather suspect that Josephine Andree would welcome a completely "solved" cube!

* * *

Our 4824 "representation challenge" ("diaLogue," October Log) created a flurry of healthy competition at New Orleans. Wayne Bradley, Chip Brawn, and Paul Greenawalt, Lincoln-Way High School, New Lenox, IL, jointly submitted solutions for 1-100, a fair feat!--but did need factorials (which we consider "non-elementary") in 29 instances.

* * *

Josephine Andree and her ingenious Cube at New Orleans convention serve to recall a somewhat parallel incident and adventure at Norman, Oklahoma convention (1983) ... the afternoon we found Mathematical Buds coordinator Harry D. Ruderman and Mathematical Log editorial team-member Diane McGowan engaged in a friendly board game of Tac-Tickle (hereunder described) in the dormitory lounge, to a full



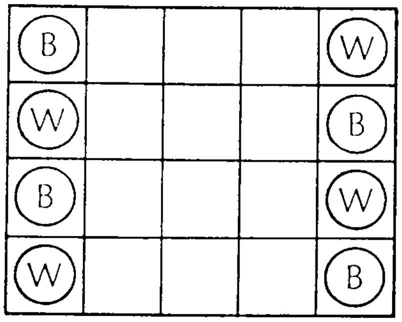
HAWAIIAN GREETINGS from the Mu Alpha Theta chapter at The Kamehameha Schools, Kapalama Heights, Honolulu (1985 convention hosts) are conveyed to New Orleans convention by chapter members Jodie Louie, Debbie Kahikina, and Heidi Takahashi. The distinctive Mu Alpha Theta license plate was spotted on the Tulane University campus.

diaLOGue

. . . FROM PAGE FOUR

gallery of youthful onlookers. "Are you sure you haven't played this before?" Harry was asking, mildly perplexed, as we, too, looked in on the contest.

Tac-Tickle, "a challenging game of strategy," comes (as marketed by Wff'n Proof), with colored pegs (red and blue) and a clever "foam" playing mat, but the essential idea, the logic, of Tac-Tickle can be appreciated by reference to a "checkers" or a pencil-and-paper version.



B				W
W				B
B				W
W				B

Tac-Tickle, a game for two, is played on a 5 x 4 grid. Four black and four white counters (let's say) are positioned in end cells, as shown. The player who goes first can choose his color. He declares it by moving one counter to an unoccupied adjoining cell. Players then take turns. Each moves one piece of his color. A move--up or down, to the right or to the left, but not diagonally--is to an unoccupied adjoining cell. There is no "jumping" or "taking." If a cell is occupied, no other piece may be moved to the occupied cell. The basis of the game, Harry explained, is to get three of one's pieces in the same line, vertically, horizontally, or diagonally, without any vacant spaces intervening. The first player to do so wins.

The Tac-Tickle rules sheet suggests interesting variations on the basic game. Tac-Tickle rules also carry, in small print, the name of the game's inventor, Harry D. Ruderman, and the date, 1965. "I developed it in California," Harry recalls. "I wanted a game you could learn quickly, and play right away." Diane McGowan--she just has natural talent, it would appear!

* * *

Harry Ruderman this Summer has seen his third Mathematical Buds collection of student papers off the press, and is welcoming possible papers for a fourth. Math Buds III is notably rich in number theoretic developments--and fascinating reading. Harry and the Log editor were making like talent scouts at New Orleans student presentations.

* * *

Seeing so many old friends at New Orleans convention served, for us, to underline one of the great strengths of Mu Alpha Theta--those busy, dedicated, and quite remarkable people (scores of them!) at national, regional, and chapter level, whose continuing interest and support (year after year!) assures the continuity upon which so much that we do depends. Also brought to mind were a unique chore that the first Mathematical Log editor, Josephine Andree, takes on so willingly and then does so well ... and two Mu Alpha Theta related book dedications, penned more than 25 years apart.

Chips from the Mathematical Log, an attractive paperback compilation of Log articles and challenges of the 1958-1965 period, and More Chips from the Mathematical Log, a companion volume of 1965-1970, remain low-price bestsellers (check national office listings of available publications). Now Josephine Andree has readied a third volume, Math to Play and Ponder, bringing her Log gleanings attractively up-to-date. The 62-page, 37-article compilation is available for $1.50 (plus 50¢ per copy for postage). Math to Play and Ponder--its charming cover art is by president-elect Betty Lichtenberg--is dedicated to the two most recent Log editors, Dr. Lichtenberg and the present Editor. Josephine Andree's dedication takes the distinctive form of a cryptarithm:

$$\begin{array}{r} \text{B E T T Y} \\ +\ \text{D O N} \\ \text{P L A Y} \\ \hline \text{P O N D E R,} \end{array}$$

with O = zero and each of the ten other letters a distinct digit, so chosen as "to make the addition true." The reader's challenge: "Two answers to honor two editors," as Josephine Andree puts it.

Being essentially a "morning person" (mathematically speaking), but fascinated to find ourselves in an Andree cryptarithm, we dived into attempted solution after a late-night popcorn social, and snarled everything hopelessly ... only to solve the cryptarithm readily after a good night's rest.

The older dedication, a long-time favorite of ours, appeared in Richard Andree's Selections from Modern Abstract Algebra (New York: Henry Holt, 1958). (Our well-worn copy provided much of our formal introduction to the spirit of "new math.") The dedication, in the form of a 2 x 2 determinant:

$$T_0$$

$$\begin{vmatrix} J_0 & -ph \\ e^{\frac{i\pi}{2}} n_e & s\, ln^{-1}\, 1 \end{vmatrix}$$

who has the acumen to help when help is needed
and has the sagacity to preserve silence when help is of no avail.

We hastened to evaluate the determinant--who wouldn't!--but hardly expected ever to find ourselves on a first-name basis with the author or with the subject of the dedication.

It's a small world, isn't it, when a shared interest, such as that in Mu Alpha Theta, is involved.

* * *

"One thing leads to another" can be as true in mathematical thinking as in any other human endeavor, as reflection on Marc Bernstein's fine paper on approximations to angle trisection (October Mathematical Log, pp. 1, 6) recently demonstrated to your Editor. The one-third part of anything can be approximated by repeated halvings, we realize. We doodled "1/3" in binary notation,

$$0.010101\ldots_{\text{two}},$$

then recast it as

$$\frac{0}{2} + \frac{1}{4} + \frac{0}{8} + \frac{1}{16} + \frac{0}{32} + \frac{1}{64} + \ldots,$$

and

$$\frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \ldots,$$

and

$$0.25 + 0.0625 + 0.015625 + \ldots,$$

becoming increasingly aware of the unanticipated pattern. Of course!--in the form

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$$

we were "summing" the infinite geometric progression, initial term 1/4, common ratio 1/4, whose sum is ... with our 20-20 hindsight! ... necessarily 1/3. (So, as a perhaps interesting alternate representation, 1/3 = 0.111... in base four.) To trisect an angle, therefore, one might divide it into four equal parts (a Euclidean construction) and take one of these, divide the next of these parts into four equal parts and add one of these, and so on, to any desired precision.

"Other bases," G.P. sums, adding smaller and smaller angles: such improbable interrelationships are what can make mathematical speculation so much fun.