

Theoretical, Experimental Combine In Demonstrations of Archimedes

By Ali R. Amir-Moéz
Mathematics Editor

Leibniz praised the genius of Archimedes, saying: "Those who knew his works and those of Apollonius marveled less at the discoveries of the greatest modern scholars." These words are justified, for Archimedes anticipated by nearly two thousand years some of the ideas of Newton and his contemporaries, and in application of mathematics to mechanics he had no equal in ancient times.

One of the Italian historians of mathematics called Archimedes "a genius more divine than human." Pliny termed him "the god of mathematics."



The work of Archimedes has one part which may be considered a combination of experiment and of the use of ideas of integration. This approach gave Archimedes the machinery for obtaining formulas for areas and volumes of many objects. In this note we would like to present a few of these ideas.

1. Circles: Accepting the fact that all circles are similar, Archimedes defines the ratio of the circumference of a circle to its diameter to be π . Then he uses integration to get the area.

Let the circumference of the circle, center O and radius r , be divided into n equal pieces (Fig. 1). Let us call each piece s_i , $i = 1, \dots, n$. If s_i is very small, we have a triangle with vertex O and base s_i whose altitude could be considered r . Thus the area of such a triangle is

$$a_i = \frac{rs_i}{2}.$$

Thus the area of the circle is very close to

$$A = a_1 + \dots + a_n = \frac{r}{2}(s_1 + \dots + s_n).$$

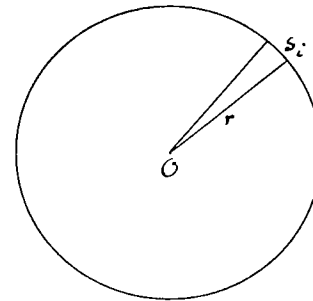


Fig. 1

When n becomes very large, we get

$$s_1 + \dots + s_n = 2\pi r.$$

Therefore

$$A = \frac{r}{2}(2\pi r) = \pi r^2.$$

(For about 212 B.C. this is a brilliant technique.)

2. Cylinders and Spheres: By techniques similar to the foregoing, Archimedes discovers the volume of a right circular cylinder of radius r and altitude h to be

$$V = \pi r^2 h.$$

We leave the analysis of the problem to the reader.

Now consider a sphere of radius r (Fig. 2). Let a right circular cylinder have radius r and altitude $2r$. We know that the volume of the sphere is

$$V_1 = \frac{4}{3}\pi r^3,$$

and the volume of the cylinder is

$$V_2 = \pi r^2(2r) = 2\pi r^3.$$

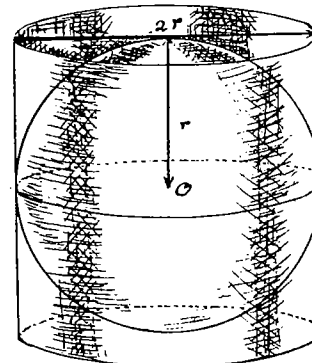


Fig. 2

But Archimedes finds V_1 by the famous experiment of submerging in water. Thus he discovers that

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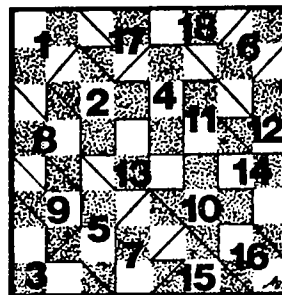
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tion that there are limits to what the editorial shoe-horn can do!
H.D.A.

back Log...

The Mathematical Log "Fractured Checkerboard" challenge of October 1982 has elicited 11 identical solutions from Mu Alpha Theta puzzle enthusiasts in seven states, adding significantly to an ever-growing list of patch-winning "Logmasters." The first such solution, here reproduced, came from Logmaster Dianne Barnes, Akron, OH--arriving on a postcard postmarked



St. Louis, prior to her school's departure from our August national convention! Other successful solvers include Susan Ellison and Terri Greene, Ocean Springs, MS, who put together the puzzle in 25 minutes from previously constructed pieces; also, Tom Hanna of Ocala, FL, John Leonard of Carlos, MN, Diane Martichuski of Irving, TX, Debby Mullett of Redstone Arsenal, AL, Janice Norton of Winter Springs, FL, Shashi Sharma of Metairie, LA, and Arlene Voorhies of New Iberia, LA.

Logmaster Spencer Greene, Spring, TX, has provided an instructive analysis of how he tackled and solved this Checkerboard Challenge. Spencer noted at the outset seven pieces possessing reflectional symmetry, three of which could not be corner pieces. On a whim he tried the other four for the corners, never "having to backtrack or change this hypothesis," as it

Mathematics is the gate and key of the sciences. ... Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy.
--Roger Bacon.

turned out. Our use of "broken squares" in this puzzle "also seemed to reduce dramatically the number of plausible configurations, thus greatly decreasing the difficulty," Spencer points out. The Editor concurs, observing however that the difficulty level proved to be about right for a good number of Log readers.

For those thriving on greater difficulty, the 50-piece Checkerboard giant of the April 1982 Log remains--significantly--unsolved!

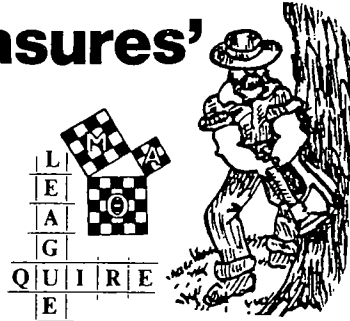
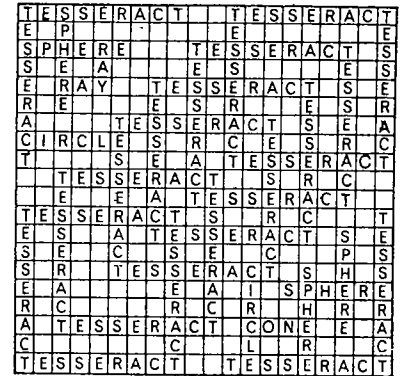
For the record, at Log press time a further Logmaster award patch goes to Spring, TX, to Truman Joe at Klein High School, for a full and careful treatment of our bowling game problem ("dialogue," December 1982 Log). More next issue!
--H.D.A.

EDITORIALY ... WITH A SHOEHORN!

Whatever our preconceived notion of editorial duties and tools of an editor's trade, the shoehorn never upstaged the dictionary and blue pencil in our initial expectations. Yet making essential components "fit" is the toughest of near-deadline Logging chores. This issue especially.

Generous-size Logs of late have assisted with the real problem of a 12- to 18-month backlog of good contributions (all editors should be so fortunate!)--but in the process have eroded budget. So Log and Tall Timbers necessarily are slimmer this time around, though not for want of first-rate material. Keep it coming by all means ... but share our growing realiza-

New Log 'Crosswords Challenge' Features 'Mingled Measures'

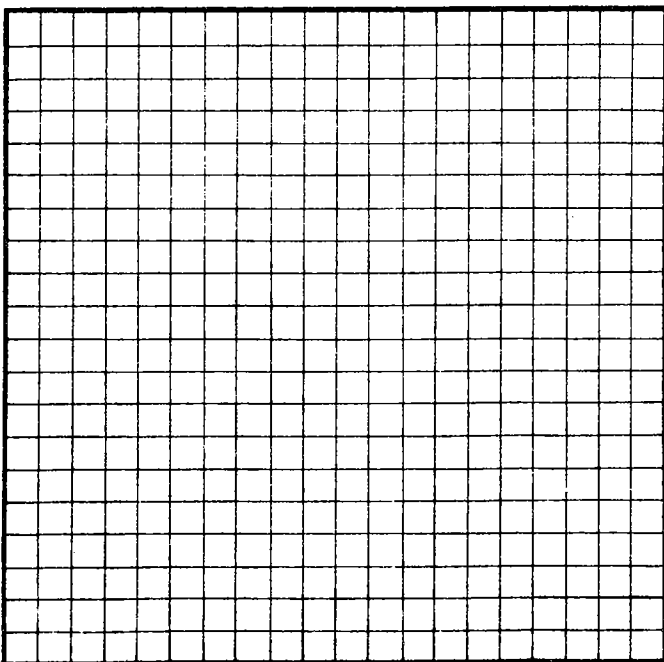


Measurement must be among the oldest and most basic of mathematics-related activities. Measurement vocabulary, accordingly, can be rich in historical interest and diverse because of the varied contexts in which measurement has taken place.

New, streamlined Système International (S.I.) metric today assumes increasing world importance. More often than not, however, new and old measures tend to mingle in these essentially transitional years.

Blending new and old in terminology, groupings, and measurement concepts, "Mingled Measures," which follows, offers a further "Crosswords Challenge" to Mu Alpha Theta word puzzle enthusiasts.

UNBEATABLE? We suspect so, the 1140-point, winning solution of Van Buren High School Mu Alpha Theta chapter, Van Buren, AR, of the Geometry "Crosswords Challenge" in the February 1982 Mathematical Log.



The thirty selected measurement words are as follows--

Units and grouping of international metric (score 5 points each time one is used):

- | | | |
|------|--------|----------|
| OHM | WATT | NEWTON |
| TEN | HERTZ | PASCAL |
| MOLE | JOULE | RADIAN |
| VOLT | KELVIN | KILOGRAM |

Older units, groupings, and concepts (score 3 points each):

- | | | |
|------|-------|------------|
| ELL | GILL | LEAGUE |
| ERG | CUBIT | PARSEC |
| ACRE | GRAIN | CALORIE |
| CORD | GROSS | CHALDRON |
| DRAM | QUIRE | HOGSHEAD |
| DYNE | STONE | HORSEPOWER |

Now the challenge and the 20x20 grid! Write in measurement words from the given lists, left to right or top to bottom, filling unoccupied squares. Allow words to "cross," much as in a crossword puzzle. Each time you enter a word, score the points associated with that particular word. Thus, acre, 3 points; two watts, 2x5 or 10 points. Not every term need be used. A term may, however, be used any number of times. The highest total score wins.

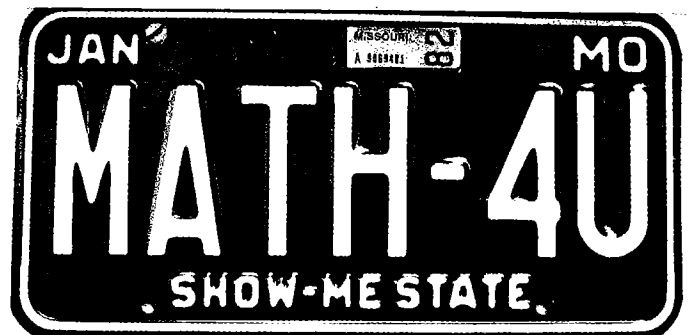
Sounds simple? It is, but to make for lively competition let's incorporate the following restrictions. Words must be "connected"--there can be no isolated

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From chaldron and cubit, hogshead and horsepower, to joule and kilogram, mole and radian, listed below are thirty measurement words--with, for purposes of our game, associated point values. The "playing field" is to be a 20x20 rectangular grid--400 unit squares. The challenge is to enter the given words, one letter to a square, according to rules (which follow), so as to achieve the highest possible point score.

Mathematics-related "Crosswords Challenges" have proven among the most popular of open competitions in recent Mathematical Logs: recall our "Famous Mathematicians" challenge (Log 25:2, Winter 1981) and "Geometric Terms" challenge (Log 26:3, February 1982), both of which elicited quite remarkable results (see "diaLogue," Fall 1981, and "backLog," December 1981 and this issue). An additional challenge to interested chapters and Mu Alpha individual members is The Log's "Mingled Measures" competition, unveiled at St. Louis and presented herewith.

Individually, competitively among friends, or as a program for an entire Mu Alpha Theta chapter, this can be fun--and also can provoke worthwhile discussion of the units, concepts, and systems involved.



MU ALPHA THETA National Convention, where else, would a license plate spotter expect to spot a license plate like this! Washington University, St. Louis, at 12th National Convention, the Editor spotted this revealing white-on-maroon plate from the "Show-Me State."

