The Mathematical Log
VOLUME 25, NUMBER 2 -- OUR 25TH YEAR -- WINTER 1981

'Fencing' Problem Generalized

by Michael W. Ecker
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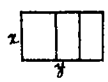
You're a farmer planning to construct a rectangular region on your land, using two partitions so as to create three "pens" as in the diagram.



You have 300 yards of fencing available. What should be the dimensions of the rectangle if the area is to be maximized?

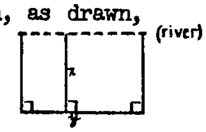
Let x, y be the dimensions of the rectangle, with x the length of a partition and A the area of the original rectangle. We must focus on the area; $A = xy$. (This focusing is important to stress, for those first encountering such a problem have a marked tendency to concentrate on other given information, such as amount of fencing, or possibly even some essentially irrelevant matter, such as the placement of the partitions.) We know in this problem that $4x + 2y = 300$; solving for y, $y = 150 - 2x$. Hence,

$$A = A(x) = x(150 - 2x) = -2x^2 + 150x.$$

One may now graph this function--but it is essential to bear in mind that y is not the dependent variable; it is A. In any case, simply locate the x coordinate of the vertex by the familiar equation for the axis of symmetry; corresponding to $y = ax^2 + bx + c$, it is given by $x = -b/2a = -150/(2\cdot -2) = 37.5$ yards. So, $y = 75$ yards, and the largest possible area is 2812.5 square yards. (Note how the use of calculus was unnecessary.)

The above leads to a nice generalization of all such fencing problems. It is possible that many of you have noticed this pattern yourselves; perhaps it is a bit of mathematical folklore.

Observe that, in the maximum area rectangle, the sum of the lengths of all sides parallel to one another is half of the fencing. This is no accident. As another illustration, if one were solving the analogous problem involving a missing side (due to presence of a natural boundary, such as a river) with one partition, as drawn,



involving 300 yards again, one would obtain a solution: $x = 50$, $y = 150$. Note how $3x = y = 150 = \frac{1}{2}(300)$.

We may summarize as follows: If a rectangular region is to be constructed utilizing m sides and/or partitions in one direction, with n in the other direction, and if f is the length of fencing available, then the rectangle of maximum area that can be so constructed has dimensions $f/2m$ by $f/2n$ (respectively). That is, $mx = ny = f/2$.

As an exercise, you may wish to prove this for yourself. Note that m and n necessarily are positive integers, and that this takes into account all possible numbers of partitions and natural boundaries. In the first example, $m = 4$ and $n = 2$, with $f = 300$; in the latter, $m = 3$ and $n = 1$, with $f = 300$.

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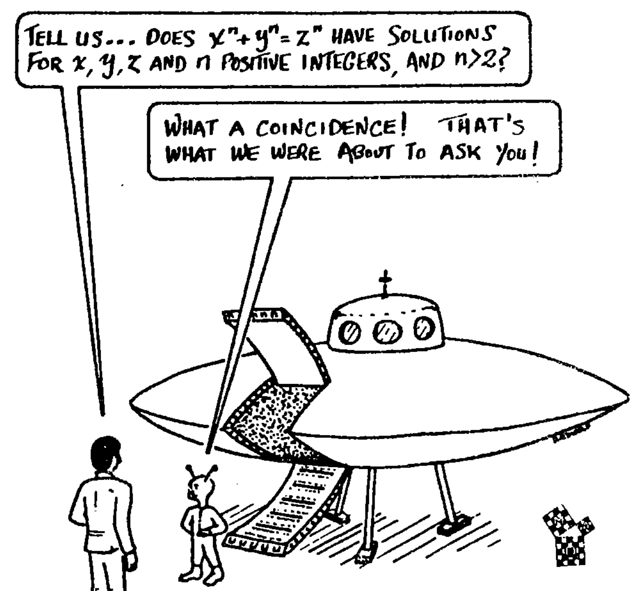

Adapted from Mathematical Digest.

The following problem was posed on the 1980 Canadian Mathematics Olympiad. It lends itself to relatively elementary insights, and may provide grist for the Mu Alpha Theta chapter mill:

If a679b is a five-digit number (in base ten) which is divisible by 72, determine a and b.

--Crux Mathematicorum.

4 COMPETITIONS 4

For chapters and individual members who enjoy the spirit of friendly competition, The Log includes four fun contests in this mid-year issue. The Editor will be pleased to learn how you made out on any or all!

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ISSN 0025-5580

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Address correspondence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman OK 73019.

Secretary's Corner

From Harold Huneke, Mu Alpha Theta Secretary-Treasurer, come the following notes and announcements:

This has been a good Fall for Mu Alpha Theta. We have chartered 34 new chapters as of January 15, 1981 and 10 petitions for charters are being processed. This Fall the Florida state Mu Alpha Theta organization mailed 250 petitions forms to schools in their state and an article on Mu Alpha Theta appeared in the October issue of the Mathematics Teacher. Both of these efforts have helped and we appreciate this type of support.

There is to be a breakfast for sponsors at the NCTM Annual Meeting in St. Louis--Friday, April 24 at 7:00 A.M. The breakfast will be in the Charles Lindberg Room of the Sheraton. The Secretary has to guarantee the number attending 72 hours in advance, so reservations are needed. If you plan to attend please let us know by April 17 (pay at the breakfast). We can always take a few latecomers, but ...

R E M I N D E R

High school students are eligible for membership in Mu Alpha Theta after they have completed four semesters of college preparatory mathematics (usually Algebra and Geometry) with a B average and in addition have completed or are enrolled in a fifth semester of college preparatory mathematics. Thus, students taking Algebra in Grade VIII could be eligible for full membership as sophomores. There is merit to involving students as early as possible. An associate membership is an option (handled locally) for students who have not completed the four semesters of college preparatory mathematics.

R E M I N D E R

The one-time initiation fee for new members of Mu Alpha Theta went to $2 as of August 1, 1980.

MATH BUDS III

Volume II of Mathematical Buds is now being typed and will be printed this Spring. Copies will be sent to chapters either this Spring or next Fall. Papers are now being solicited for Volume III. This is a unique opportunity for Mu Alpha Theta members to have works published. If you have won a prize for your paper at a Math Fair or if your teacher recommends your paper, you should send four copies to: Harry Ruderman, 2624 Division Ave., Bronx, NY 10468. Remember:

1. Title of paper
2. Author's name, address, telephone number
3. School's name and address
4. Date, place, and name of Math Fair competition at which the award was won and the nature of the award. If no Math Fair is held in your area a recommendation from a sponsoring teacher will suffice.
5. If there was a sponsor, sponsor's name, address.
6. Enclose a self-addressed envelope for returning the paper if it is not selected for publication.

SAND SCULPTURE WINS ¢A$H FOR CALIFORNIA CHAPTER

Reported by JoAnn Bertram and Peter McCauley
Bishop Garcia Diego Chapter

The scene is a crowded beach on the California coastline. Hundreds and hundreds of people are milling around mounds of wet sand, watching others digging and scratching with every kind of tool imaginable. What is it? Could it be some extraordinary phenomenon that befell the beach during the night or some Spanish gold uncovered by shifting sands? No, it is none of these strange things, yet it is still rather unusual.

It is the annual Santa Barbara County Sandcastle and Sandsculpting Contest, open to any non-profit organization in the county. The competition is for cash prizes of $200, $150, $100, and $50 for first, second, third, and fourth places, respectively.

The Mu Alpha Theta chapter of Bishop Garcia Diego High School and Bishop's Art Club jumped at the opportunity to earn money for club activities and walked away with the $150 second prize.

Armed with shovels and wheelbarrows, fifty enthusiasts hurriedly raised a five-foot (150 cm) tall, massive heap of sand, then set to the task of carving a castle from the mound. At the end of the three-hour time limit, B.G.D.H.S. students had created a masterpiece resembling, in part, the Mont St. Michel of France, yet having magnificent originality and distinctive character.

'Crosswords' Challenge

Within a chapter of Mu Alpha Theta or nationwide, there are those who like a contest, and for them The Log offers the following challenge.

Below are listed, alphabetically, twenty of the better known names from the history of pure and applied mathematics. Following each name is a score, in points. Since none of us would presume to "rate" such mathematical greats (an Archimedes vs. an Einstein!), scores are wholly arbitrary--actually, the numbers of letters in the usual English-language version of the name.

ARCHIMEDES (10)	HERON (5)
DESCARTES (9)	LOBACHEVSKY (11)
EINSTEIN (8)	MOBIUS (6)
ERATOSTHENES (12)	NAPIER (6)
EUCLID (6)	NEWTON (6)
EULER (5)	PASCAL (6)
FERMAT (6)	PTOLEMY (7)
FIBONACCI (9)	PYTHAGORAS (10)
GALILEO (7)	THALES (6)
GAUSS (5)	ZENO (4)

The challenge, briefly, is to place these names (reading left to right or top to bottom) in consecutive squares of a 20x20 grid, essentially as in a crossword puzzle. Then, each time you enter a name you score the number of points associated with that name--one Archimedes, 10 points; three Euclids, 18 points! A name may be used any number of times and it is not necessary that every name be used. Your total score is the sum of the scores associated with individual names. The highest total score wins.

The following restrictions, also part of the rules, help make for a better contest. All the names in your grid must be connected--there can be no isolated names or groups of names. Also, horizontally or vertically, adjacent squares cannot be filled unless the letters in them are contributing to one of the entered names.

The "crossword" below illustrates the intended technique, but its total score, 212, is far below the probable maximum.

```
  Z                 F I B O N A C C I
  E U C L I D   E   I
  N     O       R   B
  O     B   E R A T O S T H E N E S   D
        A       T   N                 E
  P A S C A L   O   A R C H I M E D E S
        H       S   C                 C
      F E R M A T   C                 A
    N   V       H   I       N A P I E R
  G A U S S     E       F             T
    P   K   E I N S T E I N   E       E
    I   Y   I   E       B     U   E   S
T   E       N   S       O     L   I
H E R O N   S           N     E   N   H
A           T   P Y T H A G O R A S   E
L           E           C         T   R
E     Z     I           C   Z   Z E N O
S   H E R O N     G A L I L E O   I   N
      N                     N     N
  P T O L E M Y     N E W T O N
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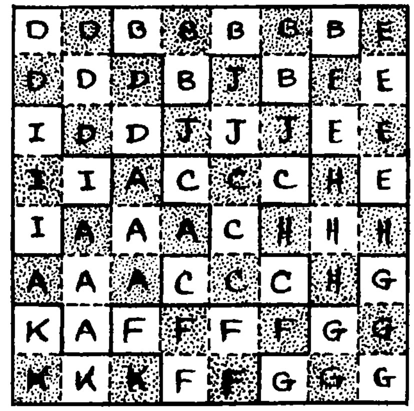
Chapters and individual members are invited to submit their best efforts. Highest "score" wins. To break ties, we'll look to the number of different names used--the more the better--then to the numbers of times the longer names are used--again, the more the better. But that's only to break a tie. The highest score wins. Have fun!

"ATLANTA CHALLENGE" READILY MET

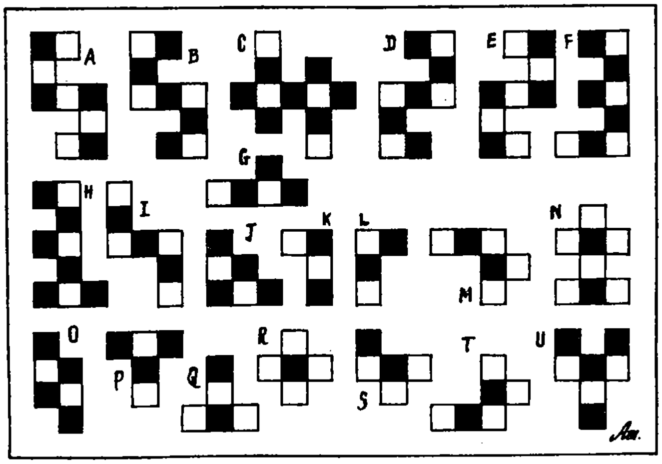


John Gmerek, a senior at St. Vincent-St. Mary High School, Akron was first with the above solution to the "Atlanta Challenge" 11-piece checkerboard dissection problem in the Fall 1980 Log. Debbie Poss, Rome, GA, supplied the identical solution, observing that her students had failed to come up with a solution essentially different from hers. Additional identical solutions have been arriving, suggesting rather strongly that the above solution may be unique. The popularity of the puzzle has led to a somewhat more demanding version being produced for this issue.

Railroad Representations...

3 6 9 2, the number on an old yellow boxcar on a siding near a school . . . but therein is a story and a challenge. For 3692, it turns out, is no ordinary number. Take a 3, a 6, a 9, and a 2, in any order; add, subtract, multiply, divide; use brackets, roots, decimal dots, dots for repeating decimals; use the numbers as digits, addends, factors, exponents . . . and see the scores of different numbers that you can represent. For example, $1 = 9 \times 3 - 26$, $2 = 9 + 2 - 6 - 3$, $3 = 69/23$, $4 = 9/2 - 3/6$, . . ., and 40 (a tough one) is $9/.2 - 3/.6$. But how high can you go? Limit yourself to signs and operations of elementary mathematics (no factorials, please), used a finite number of times. Starting with 1, how many consecutive positive integers can you represent using a 3, a 6, a 9, and a 2? Also, something harder, what are the largest and smallest positive numbers (not necessarily integers) that you can represent using a 3, a 6, a 9, and a 2, with the above restrictions? Have fun!

COMPETITION



CHECKERBOARD SUPER CHALLENGE. Shape, symmetry, logic, perseverence, and fun should combine in any systematic attack on this 21-piece dissection of an oversize 12x12 checkerboard. Cut the pieces from cardboard, we recommend. Assemble them to form a perfect 12x12 board, with alternating light and dark squares. The Editor will welcome completed solutions.

diaLOGue with the editor

Some "real-world" applications of even the most elementary mathematics can be fascinating when their principles are fully understood. Harold Jacobs, who addressed our "Georgia Tech" national convention, shows this particularly well in his textbooks, of which Mathematics, A Human Endeavor is perhaps the best known. Computer-age enumeration, complete with "check digits," can be interesting. Breaking the check digit code of an airline ticket or money order can seem a worthwhile task for an otherwise idle moment! In this spirit, George Knill provides some worthwhile reading in the January 1981 Mathematics Teacher. His "Applications" feature is on International Standard Book Numbers (ISBNs), a worldwide code to identify all new books. What the Mathematics Teacher doesn't mention (except on its title page) is that periodicals also have numbers--International Standard Serials Numbers, or ISSNs. To satisfy the spirit of curiosity (and to win the hearts of librarians everywhere), we add to our masthead (p. 2), starting this issue, our own ISSN (0025-5580) . . . in the process assuring for The Log its proper niche in worldwide bibliographic affairs.

These words are being written amid one of the heaviest snowfalls to hit this part of the country in years. With everone digging in snowdrifts, the report from Bishop Garcia Diego Chapter of creative and winning digging in sand, seems somehow appropriate. Chapter sponsor Douglas Mooers relates that this chapter has been producing a monthly newsletter by computer, and is planning a field trip to Griffith Planetarium. Our own chapter got a VIP tour of the technical side of Halifax International Airport, including the control tower, and watched jets being "talked down" on a foggy Atlantic night.

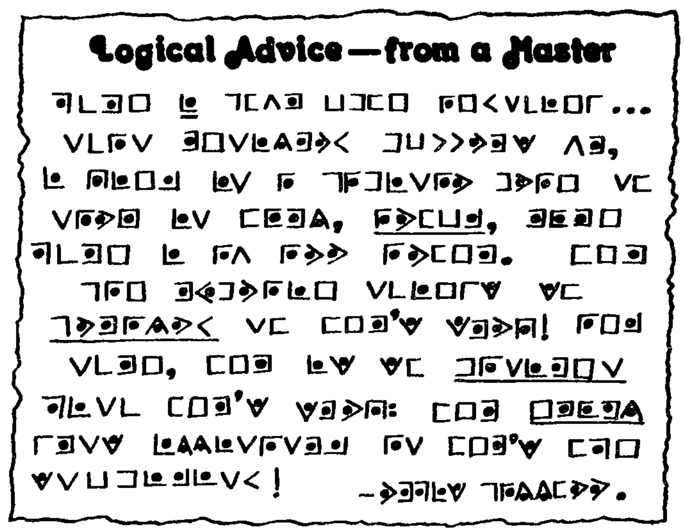
The Log takes pleasure in offering an improbable quartet of fun contests this issue. Some of them may offer "something different" for a chapter meeting. For the record, the first to report solving our sign language cryptogram, "Relativity," was Song Tan, vice-president of Southwest Miami (FL) Senior High Chapter. The key was "realizing who (Einstein) made the quote," he points out. "Do not worry about your difficulties in mathematics; I can assure you that mine are still greater," Einstein once said.

The Log routinely exchanges copies with student mathematics journals in many parts of the world. We hope to share with our readers from time to time a diversity of "exchange" items--like the delightful "space" cartoon on p. 1, adapted with permission from Mathematical Digest, published at The University of Cape Town..

Nor should we overlook the more familiar "sources" closer to home. Games magazine has come a long way, and to it we owe the idea inherent in our name-packing "crosswords" competition (p. 3). Martin Gardner in Scientific American is, for us, monthly required reading. The Journal of Recreational Mathematics puts a sound theoretical foundation to "fun and games." Then there are good general readings ("liberal arts" works on mathematics), histories of mathematics, and expository textbooks in a favored branch. Such is "grist for the mill" for the active chapter.

We most heartily recommend our sister publication, The Mathematics Student, and its superb "problems" section. Look, too, to imaginative and challenging problems in the venerable and highly respected School Science and Mathematics. What makes The Log unique, you realize, is you, the members and the chapters. Lets hear from you--about the good things at chapter level that make Mu Alpha Theta the remarkable organization we know it to be.

COMPETITION



CRYPTOGRAMS, ANYONE? Pertinent advice from one of the most delightful of logical practitioners, this "hidden message" revives an old pencil-and-paper cipher, the so-called pig-pen cipher, traditional in some secret societies. Most versions of this "code" use regular patterns of letters, and therefore are simple to "break." This version assigns symbols randomly, but even so the five commonest symbols account for over half of the characters in the message and name.