The Mathematical Log

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The Birthday Problem Extended

A well-known probability experiment concerns the so-called "Birthday Problem": In a randomly chosen group of n persons, what is the probability that at least two persons share the same birthday anniversary?

To solve this problem, the probability that no two people share the same birthday is first caluclated. If the n persons were arranged in some order and each of them was to declare his/her birthday, then 365^n different n-tuples are possible (ignore February 29). If all birth dates are considered to be equally likely, each of these 365^n-tuples is equally likely as well.

To find the number of n-tuples which contain no matches, again visualize the n people declaring their birth dates in order. The first person may declare any of 365 days. To avoid a match, only 364 dates are available to the second person; 363 are available to the third person; 362 to the fourth person. The n^{th} person has (365-n+1) or 366-n dates available. The number of ways in which the n people can fail to have a match is (365)(364)(363)(362). . . (365-n+1). The probability of **no** matches is thus

$$\frac{(365)(364)(363)(362)\ldots(365\text{-}n\text{+}1)}{365^n}$$

The probability of **at least** one match is then

$$1 - \left[\frac{(365)(364)(363)(362)\ldots(365\text{-}n\text{+}1)}{365^n}\right]$$

Surprisingly few people are needed in a group in order that the probability of at least one match exceeds 1/2. For 22 people, the probability of at least one match is

$$1 - \left[\frac{(365)(364)(363)\ldots(344)}{365^{22}}\right] = 1 - .5243 = .4757;$$

however, the probability of at least one match for 23 people is

$$1 - \left[\frac{(365)(364)(363)\ldots(343)}{365^{23}}\right] = 1 - .4927 = .5073$$

Let 23 be the critical value in the Birthday Problem; that is 23 is the smallest number of persons for which the probability of at least one match exceeds 1/2.

A similar experiment can be performed with decks of 52 regular playing cards. If each of a group of n-persons, draws a card from his/her deck, what is the probability that at least two of them will get exactly the same card (same suit, same value)? The same reasoning as that used in the Birthday problem shows that the probability of at least one match with n people is

$$1 - \left[\frac{(52)(51)(50)\ldots(52\text{-}n\text{+}1)}{52^n}\right].$$

If n = 8, the probability of at least one match is .4325 while if n = 9, the probability of at least one match is .5193. The critical value for n is therefore 9.

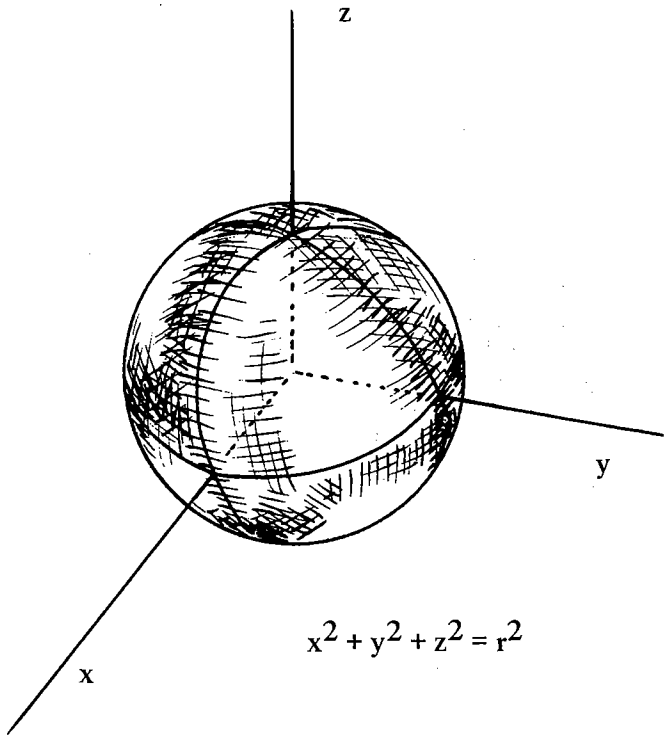
Let us generalize the problem. Suppose that each of n persons has a set of cards numbered consecutively from 1 to r. If each person selects a card at random from his/her deck, what is the probability of at least one match? The same reasoning as that used in the two previous examples leads to the conclusion that the probability of at least one match is

$$1 - \left[\frac{r(r\text{-}1)(r\text{-}2)\ldots(r\text{-}n\text{+}1)}{r^n}\right].$$

For a given value of r, what is the critical number for n; that is, what is the smallest value for n so that the probability of at least one match exceeds 1/2? Table I depicts critical values for n.

(continued on page 4)

The Equation Of A Basketball



$$x^2 + y^2 + z^2 = r^2$$

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Secretary's Corner

During the period from July 1, 1979 to January 15, 1980, 8,616 students have joined Mu Alpha Theta and 37 new chapters have been approved. Four charters are pending approval. We are alive and well.

We did not send the Instructor's Manual for *Cryptarithms* or *Logic Unlocks* to the chapters when we sent out the booklets. These manuals are available from the Mu Alpha Theta office. The cost to Mu Alpha Theta members is $1.20 each.

We regret to announce the death of Dr. Carl Olds, Professor-Emeritus of Mathematics at San Jose State University. Professor Olds was the first editor of *The Mathematical Log.*

Mu Alpha Theta is sponsoring a session on Mathematics Competitions at the Fourth International Congress on Mathematics Education to be held at Berkeley, California in August 1980.

State or regional meetings we know about are as follows. **Texas** — February 8-9 at San Antonio; **Tennessee** — March 5-8 at Smyrna; **Florida** — February 15-16 at Brandon; **Philadelphia** — May 23-24 — Regional; **Oklahoma** — October 14 at Norman. Would others planning meetings please notify our office so that we may publish your dates and locations?

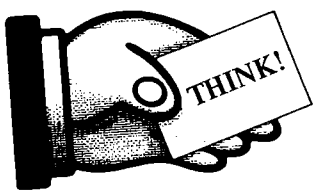
Students at Bronx High School of Science publish each year, *The Math Bulletin.* Their goal is to provide better communication in mathematical knowledge among students, parents and others. For information, write: Abby Amsterdam, Editor-in-chief, Math Department, The Bronx High School of Science, Bronx NY 10468.

Announcements

The annual breakfast for sponsors will be held at 7:30 a.m. Friday, April 18, at the National Council of Teachers of Mathematics meeting in Seattle, Washington. It will be held in the **Colonial Room** of the **Olympic Hotel** and the cost will be **$4.95 plus gratuity and tax.** Send reservations, no money, to Harold Huneke at the National Office.

Two other publications in the series developed by Richard and Josephine Andree are available from the Mu Alpha Theta office.

Secret Ciphers $2.50
Secret Ciphers plus the Instructor's Manual $3.50
Solving Ciphers $2.50
Solving Ciphers plus the Instructor's Manual $3.50



A WIT—TWISTING ARITHMETICAL APPTITUDE TEST

1. If you went to bed at 8:00 at night and set the alarm to get you up at 9:00 in the morning, how many hours of sleep would you get? _____

2. Do they have a Fourth of July in England? _____

3. Why can't a person living in Winston Salem, North Carolina, be buried west of the Mississippi River? _____

4. How many birthdays does the average man have? _____

5. If you have only one match and entered a room in which there were a kerosene lamp, an oil burner, and a wood stove, which would you light first? _____

6. Some months have 30 days and some have 31. How many have 28 days? _____

7. If a doctor gave you three pills and told you to take one every half hour, how long would they last? _____

8. A man builds a house with four sides to it and it is rectangular in shape. Each side has a southern exposure. A big bear wanders by. What color is the bear? _____

9. How far can a dog run into the woods that is seven miles in diameter? _____

10. What is the minimum number of active players on a baseball field during any part of an inning? _____

11. I have in my hand two United States coins which total 55¢ in value. Please bear in mind that one is not a nickel. What are the coins? _____

12. A farmer had 17 sheep. All but 9 died. How many did she have left? _____

13. Divide 30 by 1/2 and add 10. What is the answer? _____

14. Two people are playing checkers. They play five games and each person wins the same number of games. How do you figure this? _____

15. Take two apples from three apples and what do you have? _____

16. An archaeologist claimed he found some gold coins dated 46 B.C. Do you think he did and why? _____

17. A woman gave a beggar 50¢. The woman is the beggar's sister but the beggar is not the woman's brother. How can this be? _____

18. How many animals of each species did Moses take on the ark with him? _____

19. Is it legal in North Carolina for a man to marry his widow's sister? _____

20. If a plan crashed on the border of Mexico and the United States, where would they bury the survivors? _____

Bonus Question: What word is misspelled in this test? _____

Fill out your answers and send them to the Editor. We'll see the results in the next issue of The Mathematical Log.

1980 Convention Information

We hope that you're making plans to attend the Tenth National Convention of Mu Alpha Theta in Atlanta, Georgia, August 3-6. Several general sessions are being planned, including one for Sunday evening after which we will have the first round of the competition.

Some of the speakers for the general sessions will be: Dr. Jack Downes of Georgia State University, Dr. John Neff of Georgia Tech, and Dr. Tom Thomson of Kennesaw College.

In addition to the general sessions, there will be numerous section meetings, student presentations and the mathematics competition. On Tuesday afternoon, we'll go sightseeing in Atlanta on our way to Six Flags Over Georgia.

Each school that pre-registers will be sent a copy of the regulations concerning conduct at the convention.

Housing: On the campus of Georgia Institute of Technology, Atlanta, Georgia. Separate air-conditioned dormitories for girls and boys with two students assigned to a room. $1.00 refundable key deposit.

Costs: $51.00 per person for three nights lodging (linens included except pillows). Nine meals (Sunday night through Wednesday noon and Six Flags outing).

Pre-Registration Fees: $3.00 for members and sponsors, $5.00 for non-members.

For further information contact:
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The Birthday Problem Extended

(continued from page 1)

TABLE I

r (number of cards held by each person)	Critical value for n (Smallest number for which the probability of at least one match exceeds 1/2)
2	2
3 - 5	3
6 - 9	4 (Note r = 6 is the same situation as if each person rolled one hexahedral die and r=8, if each person rolled one octahedral die)
10 - 16	5 (r=12 is the same situation as if each person rolled one dodecahedral die)
17 - 23	6 (r=20 is the same situation as if each person rolled one icosahedral die)
24 - 32	7
33 - 42	8
43 - 54	9
55 - 68	10
69 - 82	11
83 - 99	12
100 - 116	13
117 - 134	14
135 - 156	15
157 - 178	16
179 - 201	17
202 - 226	18
227 - 252	19
253 - 280	20
281 - 309	21
310 - 340	22
341 - 372	23

As an example of how to read Table I, suppose that each person in a group draws a card at random from a set of cards numbered 1 through 250. Since, for r = 250, the critical value of n is 19, the probability of there being at least one match will exceed 1/2 if there are 19 or more persons in the group. If there are 18 or fewer persons in the group, the probability of at least one match is less than 1/2.

Challenges For The Reader:

1. Verify the critical values of n in Table I by direct calculation.
2. Extend Table I to at least r = 1000.
3. Table II reports the number of values of r associated with each critical value of n.

TABLE II

Critical Value of n	No. of Values of r
2	1
3	3
4	4
5	7
6	7
7	9
8	10
9	12
10	14
11	14
12	17
13	17
14	18
15	22
16	22
17	23
18	25
19	26
20	28
21	29
22	31
23	32

Conjecture the rate at which column 2 will increase using your results from Challenge 2.

Conjecture the rate at which column 2 will increase using your results from Challenge 2.

4. Program a computer to reproduce and extend Table I indefinitely with no need for operator input during the program.

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MATH MIXT-MAXIMS

Make up more Mathematical Mini-maxims. The best ones we receive will be published in future issues of *The Mathematical Log.*

Answer: Pie In The Sky!