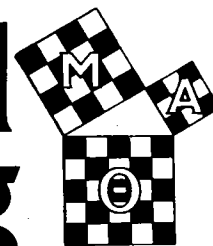


# The Mathematical Log

VOLUME XXIII, NO. 2 – WINTER 1979

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## PATTERNS

Quite frequently mathematical solutions are extracted from pieces of data by first looking for a pattern, making a conjecture based upon the observed pattern and then using the algebra necessary to prove or refute the conjecture. Let's look at three examples that fit this model.

### Example 1.

Does the following generate a pattern?

$$5^2 - 5 = 4^2 + 4$$

$$7^2 - 7 = 6^2 + 6$$

Usually this causes a consideration of what would be the next example to continue the possible pattern, some saying  $6^2 - 6 = 5^2 + 5$  while others claim  $9^2 - 9 = 8^2 + 8$ . In either case, the statement is true as is any other example of this type that may have been tried. This leads us to believe it would be true for any example. Algebraically we can see: If  $a$  and  $b$  are natural numbers,  $a^2 - a = b^2 + b \iff a^2 - b^2 = a + b$

$$\iff (a-b)(a+b) = a + b$$

$$\iff a - b = 1.$$

But  $a - b = 1$  was the condition of our possible pattern, hence, there is indeed a pattern.

### Example 2.

Note: (a)  $2^2 + 3^2 = 13$ , and  $12 + 2^2 = 5$ ;  $13 \cdot 5 = 65 = 7^2 + 4^2$  and (b)  $3^2 + 4^2 = 25$  and  $2^2 + 5^2 = 29$ ;  $25 \cdot 29 = 725 = 25^2 + 10^2$ . Is it always true that "a product of sums of two squares is a sum of two squares?"

After being sure one understands the question, we generate further cases and check the results:  $1^2 + 3^2 = 10$ ,  $2^2 + 1^2 = 5$ ,  $10 \cdot 5 = 50 = 7^2 + 1^2$ , which holds as does  $5^2 + 7^2 = 74$ ,  $6^2 + 3^2 = 45$ ,  $74 \cdot 45 = 3330 = 37^2 + 9^2$ . We also note that with larger numbers involved, finding the last sum of two squares is fairly complicated when using only a "squares" table or even a calculator.

Using variables we demonstrate that  $(a^2 + b^2) \cdot (c^2 + d^2) = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 = a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2$ ,

and using the "add zero" trick we find  $= a^2c^2 \pm 2abcd + b^2d^2 + b^2c^2 \mp 2abcd + a^2d^2 = (ac \pm bd)^2 + (bc \mp ad)^2$

which verifies that this is a pattern. Furthermore, under appropriate conditions, there are two distinct ways to find the last sums of two squares.

### Example 3.

Does the following generate a pattern?

$$(20 + 25)^2 = 2025$$

$$(30 + 25)^2 = 3025$$

To gather evidence concerning the possible pattern we consider  $(40 + 25)^2 = 4225 \neq 4025$ ,  $(50 + 25)^2 = 5625 \neq 5025$ , and  $(60 + 25)^2 = 7225 \neq 6025$ , which allows us to refute this as a pattern. However, if interested in whether any further examples of this type work, we look algebraically and find, if

$$(10a + 25)^2 = (10a) \cdot 100 + 25$$

$$\implies 100a^2 + 500a + 625 = 1000a + 25$$

$$\implies 100a^2 - 500a + 600 = 0$$

$$\implies a^2 - 5a + 6 = 0$$

$$\implies (a - 3)(a - 2) = 0$$

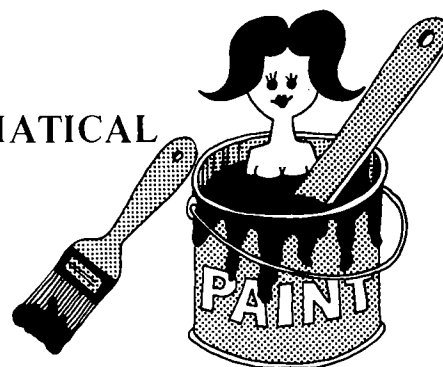
$$\implies a = 3 \text{ or } a = 2,$$

which shows the two examples with which we started are the only two that work in the manner suggested.

From the three examples cited, we see that possible patterns lead mathematicians to examine similar cases to check the possibility of the pattern existing. If no counter example is quickly forthcoming, then abstract manipulations are usually performed, which may verify the pattern, or exhibit the constraints under which the possible pattern holds.

Melfried Olson  
University of Wyoming  
Laramie, Wyoming 82071

## PROBLEMATICAL PALETTE



A very tiny drop of blue paint is (theoretically) placed on each rational number on the real number line. A very tiny (the same size) drop of yellow paint is placed on each irrational point on the real number line. The colors mix. What is the color of the thus-painted real number line? Answer on page 3.



## WINNING AT DARTS

A Canadian carnival game (Acme Amusements Ltd., "Add 'em Up Darts") features an unusual "dartboard" consisting of a rectangular grid of numbered squares. The player has 4 darts, all of which must hit the board, and each square scores 1 to 6 points. Numbering, denoting scoring, is as follows:

```

2 4 3 4 2 4 3 4 3 5 2 4 3 5 2 4 3
4 3 5 3 6 1 5 3 4 3 4 5 4 3 4 6 2
2 4 3 4 3 4 3 6 5 4 3 4 3 2 1 3 5
3 2 5 3 4 2 4 3 1 2 5 3 4 3 5 4 2
2 4 3 5 3 5 3 4 3 4 2 5 3 6 2 3 4
5 3 4 3 4 3 6 2 4 3 4 1 5 3 4 2 3
3 4 2 4 1 5 4 3 5 4 3 4 3 2 3 4 5
4 5 3 5 4 3 5 2 3 1 3 4 6 4 2 3 4
2 3 4 3 2 4 2 3 4 3 4 2 3 2 5 4 3
3 5 2 4 3 1 3 5 3 4 3 6 4 3 2 3 4
2 3 4 2 5 3 4 3 2 5 4 2 3 4 3 5 2
4 1 5 4 3 4 6 2 4 1 3 4 5 3 4 2 3
3 6 3 2 5 3 2 4 3 4 5 3 4 1 2 3 4
4 3 2 3 4 5 4 2 4 3 2 4 2 3 5 4 2

```

"14 WINNERS. . . 7 LOSERS," declares the carnival board, tabulating that total scores of 10-16 lose (7 possibilities), while the other 14 (4-9, or 17-24) "win". More specifically, 9 and 20 win a "small prize," 6,7,8,19, 21 and 22 win a "big prize," 4, 5, 23 and 24 give the player a choice of prizes, and 17 and 18 are "special" (earning a free game, very possibly).

1. Player A always hits the board, but hits the numbers more or less by chance.

- Consider A's possibility of a total of 24. More than one dart can hit a square.
- Consider A's probability of winning a choice of prizes in one game.
- Consider A's probability of a win.

2. A particularly good shot, Player B hits the square aimed at 50% of the time, and one of the 8 surrounding squares (with roughly equal probability) the other 50%. B sets out to win by shooting 6's. What are B's chances of a choice of prizes? . . . of a win?

H.D. Allen  
Nova Scotia Teachers College  
Truro, Nova Scotia, Canada

## ANNOUNCEMENTS

### Attention Region 3. . .

At the national meeting in Stevens Point this summer, your delegates asked to have a question exchange established. That seems to be the biggest problem in setting up math contests, math bowls, math fairs, etc. My chapter has agreed to try. Please give us plenty of lead time. Questions for math bowls should be on transparencies. Here are the rules and we hold the option of changing them if there are difficulties.

1. Every request for **one** question must be accompanied by at least **two** sets of questions. They do not have to be the same type as requested (math fair, math bowl, etc. We need to build up a supply.).

- Each set of questions must have attached to it
  - solution
  - rules for that particular event
  - time of the school year it was used
  - group using them (8th grade, all schools, etc.)

3. Include a large brown stamped, self-addressed envelope. Send your requests to:

Mu Alpha Theta Question Exchange  
Milwaukee Technical High School  
319 West Virginia  
Milwaukee, Wisconsin 53204

Happy contests to us all.

Adele Hanson  
Governor Region 3

### LETTER TO THE EDITOR

Dear Math Log:

Everyone knows that great English writer, Bill Shakespeare, was a Trig whizz. We awakened to this fact while thumbing through his play *Macbeth*.

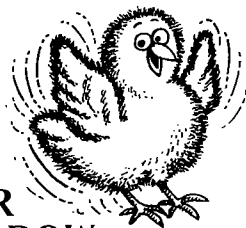
Below is our interpretation of this famous excerpt, from Act V, Scene V:

#### Macbeth's Soliloquy With Respect To Trig

*Tomorrow, and tomorrow, and tomorrow  
Creeps in these petty tests from day to day  
To the last syllable of recorded grades;  
And all our yesterdays have lighted cosines  
The way to dusty death. Out, out brief 4 point!  
Life's but an approaching asymptote, a poor parabola,  
That stretches and curves its hour upon the graph  
And then is heard no more. It is a formula  
Devised by an idiot, full of exponents and variables,  
Signifying nothing.*

We hope you enjoyed reading it as much as we enjoyed writing it.

B.J. Magness  
Karen Shirley  
Hot Springs, Arkansas 71901



## CALIFORNIA CHAPTER REACHES FOR TOMORROW

The Bishop Garcia Diego Chapter of Mu Alpha Theta sponsored "Reach For Tomorrow", a day of solar energy displays and classroom presentations. The idea was formulated by Miria Mina and Jolyne Bauer, both seniors.

The objective was to stimulate student awareness and interest in the direction of the future by stressing the importance of long range planning and intelligent considerations of alternative lifestyles and energy development.

The Chapter contacted several community resources and put together various in-class presentations which took place hourly. They included: "To Live On Another World" presented by the California Museum of Science and Industry; "Liquid Natural Gas", presented by the Southern California Gas Company; "Recycling: A Need for Conservation" presented by the Santa Barbara Ecology Center; and "Tomorrow's Technology Today" presented by Delco Electronics Corporation of Santa Barbara.

In addition to the classroom presentations, numerous outdoor displays were placed on view during the lunch break. Among them were the following and the organizations which sponsored them:

Observation of the Sun	SB Astronomy Association
Solar Oven	Sunrae Corporation
Missile Launching	U.S. Air Force
The "Lotus" Automobile	Santa Barbara Imports
Micro Chip Technology	Delco Electronics
Home Energy Caluclators	U.S. Department of Energy
Solar Water Heater	Santa Barbara Solar Energy Systems
Solar Collectors	Sunergy Corporation
Ecology Display	Sierra Club
Collector Plates	Univ. of California, S. Barbara

Also held was a writing contest with a wide range of themes concerning the future. The papers were judged by the English department with cash awards to the first three winners. Several students earned extra credit for their work on "Reach For Tomorrow" projects.

Chapter President Dawn Bunker received congratulations from Principal Sr. Madeline Clancy and many faculty members for the project. The club sponsor is Mr. Douglas Mooers.

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Address correspondence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman, Oklahoma 73019.

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**President-Elect:** Katherine P. Layton, Department of Mathematics, Beverly Hills High School, Beverly Hills CA 90212.

**Secretary-Treasurer:** Harold V. Huneke, University of Oklahoma, 601 Elm Avenue, Room 423, Norman OK 73019

**Governors:** James T. Woolum, Clayton Valley High School, 1101 Alberta Way, Concord CA 94521; Kathryn Hisenbrock, Community High School, Charles City IA 50616; Adele Hanson, Milwaukee Technical High School, 319 West Virginia, Milwaukee WI 53204; Thomas Thrasher, Austin High School, Danville Road SW, Decatur AL 35601.

**Editor:** Dr. Betty Lichtenberg, Department of Math Education, University of South Florida, Tampa FL 33620.

**MAA Representative:** Prof. Robert Wilson, Math Dept., Ohio Wesleyan University, Delaware OH 43015

**NCTM Representative:** Kathryn Fleischman, 353 Siegfried Drive, Buffalo NY 14221

## 1979 CONVENTION PLANS ARE ROLLING!

Plans for the 1979 Mu Alpha Theta Convention to be held in Athens, Alabama at Athens State College are well underway. The first letter containing information about the Convention has been mailed and a second letter is coming shortly.

Sponsors. If you have not received any information, write to Tom Thrasher, Austin High School, Decatur Alabama 35601 and get on the mailing list.

The Convention will be held August 5-8.

## PALETTE SOLUTION

Green you say? No!

Ah yes. It is true that blue and yellow do make green, when properly mixed in like amounts. But there are so many more irrationals than rationals that we have to use so much more yellow paint — an infinite amount, in fact — that the painted number line is totally and completely **YELLOW!** The cardinality of the rationals is aleph null; that of the irrationals is  $C_1$ . Do a bit of reading on your own on these two infinite sets.

**Cletus Oakley**  
Haverford College  
Haverford, Pennsylvania 13041



## MU ALPHA THETA — SINCE 1957

When I began teaching in Newark in the fall of 1957, I found our school devoid of any kind of mathematics organization. Having sponsored a mathematics club in the school where I taught previously, I began to advocate a similar interest among my students there. Coincidentally, the national high school and junior college mathematics club, Mu Alpha Theta, had been conceived shortly before that time by a committee of the National Council of Teachers of Mathematics under the chairmanship of Dr. Richard Andree of the University of Oklahoma. As a matter of fact, a group of mathematicians drew up a constitution in the fall of 1956 and the club began formally in the spring of 1957. Therefore, our local mathematics club was organized at an appropriate time to apply for affiliation and, hence, became a charter chapter of the national organization.

In its nearly twenty years at Newark Senior High School, Mu Alpha Theta has promoted interest in and enjoyment of mathematics among our academic students. Several hundred outstanding mathematics students have contributed to the success of our chapter and have benefited from its non-classroom mathematical activities.

Any student in Newark Senior High School who has completed or is now enrolled in his third year of college preparatory mathematics, who has a 3.3 average in mathematics, and who has a 3.0 average in all subjects, is eligible for membership. These qualifications are checked twice each year, and students who meet the standards receive written invitations to join our group. Not all students who qualify accept our invitation, but those who join manifest their interest through regular attendance for at least a semester, sometimes almost an entire year, before they attain full membership. Those students who meet scholastic and attendance requirements are initiated each year at a formal ceremony to which parents are invited. Initiates receive a certificate and membership card which then entitles them to full privileges of the organization.

Each year our school inducts twenty-five to thirty-five new students into Mu Alpha Theta, which is a subset of our Mathematics Club. This group, along with former initiates, constitutes more than half of our membership which usually approximates sixty. Our constitution is written to include both groups, but requirements for membership in our Mathematics Club are less rigid. These students must complete the same number of years of academic mathematics, but they need only to show passing grades to join.

Our chapter meets regularly two times each month at which times a business meeting and a program are held. The president

conducts the business meeting at the outset, and the vice president, who serves as program chairman, introduces the program. Our members sometimes select, research, and report on a topic not taught in the classroom; speakers come to us from colleges, universities, businesses, and industries to discuss pure and applied mathematics. Our club sponsors, at least twice each year, tours of local businesses and industries which use mathematics extensively in their operations. Special features of our annual program include a joint meeting with the Newark Council of Teachers of Mathematics, and Mathematics Recognition Night, at which time all students in the mathematics department who have won special awards during the year are honored for superior achievement. Our chapter sponsors an annual cash award in mathematics which goes to that senior who achieves highest excellence in his or her class in mathematics and who plans to pursue the study of mathematics or a mathematics-related field. Our only two social events of the year are the Christmas party and the spring picnic to honor our senior members.

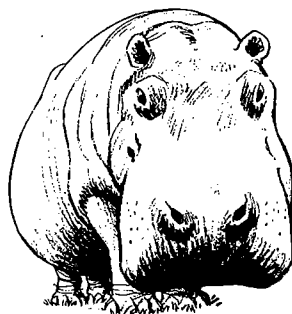
Our club encourages its members and other mathematics students to participate in the Statewide Mathematics Contest sponsored by the Ohio Council of Teachers of Mathematics and in the Annual High School Mathematics Contest sponsored jointly by the Mathematical Association of America, the National Council of Teachers of Mathematics, Mu Alpha Theta, and the Society of Actuaries.

During the years which our club has flourished, the Newark Chapter has hosted two regional meetings which attracted students from chapters in Ohio, Pennsylvania, and West Virginia. On the occasion of the tenth anniversary of our charter, our group held a reunion banquet to which all alumni of the chapter for the previous ten years were invited.

To raise money to operate, our members have sponsored dances, car washes, and candy sales. The sale of candy has been the most profitable and the most satisfactory means of meeting our expenses.

My years as sponsor of the Newark Chapter have been rewarding since I have seen many students gain a deeper appreciation for and enjoyment of mathematics through the activities of our organization. If you are already the sponsor of a chapter, you know and understand that of which I speak; if you are not, I encourage you to establish a chapter.

Wilbur Powers  
Newark Senior High School  
Newark, Ohio



ABSOLUTELY | SILLY |

A German ordering a mug of beer says. . .

**Einstein!**

The non-Mu Alpha Theta student's response when asked if he got an A in Geometry. . .

**R-Euclidean!**

**PLEASE SEND IN JOKES — PLEASE SEND IN JOKES**