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Variations On Magic Squares

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This note intends to give some variations on well-known magic squares rather than how to obtain them. One may find quite a bit of theory in [1]. We shall show how one can construct magic squares from consecutive terms of an arithmetic progression.

1. **Three-by-three Squares:** Most everyone knows how to get the magic square of (Figure 1) by trial and error.

2	9	4
7	5	3
6	1	8

Figure 1

We'd like to say a few words about this particular magic square and give some variations on it. In order to start a nine-number magic square, we write 0 (zero) in the center square (Figure 2).

1	2	3
4	0	4
3	2	1

Figure 2

Then we write 1, 2, 3, 4, 1, 2, 3, 4 as in the Figure 2. Next we change some of the opposite numbers to their negative (Figure 3).

1	2	-3
-4	0	4
3	-2	-1

Figure 3

This way if we add numbers of each row or each column or each diagonal, we get zero.

From this magic square we get many others by adding a fixed number to every one of the numbers in the square. For example, by adding 5 we get Figure 1 slightly turned around (Figure 4).

6	7	2
1	5	9
8	3	4

Figure 4

By adding another 5 we get (Figure 5).

11	12	7
6	10	14
13	8	9

Figure 5

We observe that the numbers used in each magic square are **consecutive.** For example, in Figure 5 we have 6, 7, 8, 9, 10, 11, 12, 13, 14. Thus, with any nine consectuive integers one may construct a magic square. Indeed, one may use nine consecutive elements of an arithmetic progression with common difference **one.**

Now we shall look into another variation. If we choose any of the magic squares which have been considered so far, and multiply all elements of it by a fixed number **d,** we obtain another magic square. For example, if we multiply elements of Figure 4 by **2,** we obtain (Figure 6).

12	14	4
2	10	18
16	6	8

Figure 6

One observes that this magic square has been constructed from the first consecutive **even** natural numbers. Thus, one can construct a magic square using **nine** consecutive elements of any **arithmetic progression.**

Here we give more details. Let a+d, a+2d, a+3d, . . ., a+9d be nine consecutive terms of an arithmetic progression. Then we consider,

$$a/d + 1, a/d + 2, \ldots, a/d + 9$$

It is clear that adding a/d to elements of the square of Figure 4 gives a magic square. Then we multiply every element of this square by d and we obtain the magic square constructed from the terms of the progression.

2. Sixteen Consecutive Elements: The theory of magic squares is an interesting part of number theory. Since we would like the article to be elementary, we only give a few magic squares without explaining how they are obtained. For a rigorous treatment of the subject see [1].

Now we shall give a magic square of sixteen consecutive integers (Figure 7).

(Continued on Page 4)

Secretary's Corner

Some items of interest from the Secretary's Annual Report are:

- Last year 80 new Mu Alpha Theta Chapters were chartered and 18,603 new members were added to our rolls.
- Dr. Robert Wilson of Ohio Wesleyan University was reappointed as of January 1979 for another three-year term as the representative to our Governing Council from the Mathematical Association of America. Mr. Alvin Gloor of Omaha, Nebraska was appointed as of May as the representative to our Governing Council from the National Council of Teachers of Mathematics.
- Another Mu Alpha Theta publication, *Magic Unlocks,* written by Dr. and Mrs. Andree, will be sent to chapters along with the fall mailing of the *Mathematical Log.* Additional copies are available from the Mu Alpha Theta office for $1.20 to Mu Alpha Theta members or sponsors.
- *The Instructor's Manual for Cryptarithms* is available from the Mu Alpha Theta office at a cost of $1.20 to members.
- Rising costs for postage, printing, secretary, publications, etc., have caused us to dip into our savings account. As a result, the Governing Council is recommending raising the initiation fee from $1 to $2.00. This will be voted on during the spring of 1980 and more information will be provided at that time.
- The Governing Council noted with regret the death of Julius Hlavaty, a former president of Mu Alpha Theta and the National Council of Teachers of Mathematics. The Council voted to honor Julius by dedicating Volume II of *Mathematical Buds* to him.
- State and regional Mu Alpha Theta meetings that were reported include: Florida at Coral Springs High School; Texas hosted by the Keystone High School Chapter in San Antonio; Mississippi hosted by the Gulfport High School Chapter; Tennessee hosted by Briarcrest Baptist High School, Memphis; and, Wisconsin hosted by Brookfield East Chapter, Milwaukee.
- The second Student Delegate Assembly was held at the National Convention. Delegates elected for 1979 were: **Region I** - Steven Myers, 8266 Grand Ave. NE, Bainbridge Island WA 98110; **Region II** - Brenda Martin, 737 Elmeer Ave., Metairie LA 70005; **Region III** - David Marmaduke, 1027 Princeton St., Akron OH 44311; **Region IV** - Linda Carole Porter, 6401 Wood Bridge Rd., Memphis TN 38138.

As a result of recommendations from the Student Delegate Assembly, a copy of the questions used in the mathematics competition at the national convention will be sent to each chapter this fall.

Math Buds

The papers for Volume II of Math Buds have been selected and are now being edited. Volume II will probably be sent to chapters in the summer or fall of 1980.

Papers are now being solicited for Volume III. This is a great chance for high school or junior college students to get their work published.Please send your papers to : Harry Ruderman, Hunter College High School, 94th St. & Park Ave., NY,NY 10028.

When a paper is submitted, the following information should be given on a separate sheet: (1) title of paper, (2) Author's name, address and phone number, (3) school's name and address, (4) date, place, and name of Math Fair competition at which the award was won and the nature of the award (5) if there was a sponsor, his/her name and address.

Elections

Elections will be held in the spring of 1980 for the following offices in the Governing Council: President-Elect, Secretary-Treasurer, Governor Region I (Western), Governor Region II (Central). President Katherine Layton has appointed the following nominating committee: Dr. Robert Kalin (Chair), Mr. Alvin Gloor, Dr. Robert Wilson. Chapters or individuals are invited to submit names to the committee on or before December 15, 1979 along with appropriate biographical information on the potential candidate. Please send nominations to : Dr. Robert Kalin, 1120 Cherokee, Tallahassee, FL 32306.

Sponsors Breakfast

There will be a breakfast for sponsors on Friday, April 18, 1980 at the NCTM meeting in Seattle, Washington. Send reservations to Harold Huneke at the National office.

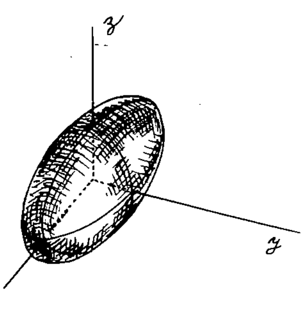
Next National Convention

Start planning now to attend the next national convention August 3-6 on the campus of Georgia Tech, Atlanta, GA. We are really interested in student papers so you may wish to start work on one this fall.

You may have already written to Pam Drummond for information, but just in case: Pam Drummond, George Walton High School, 1590 Bill Murdock Rd., Marietta, GA 30062.

The Equation Of A Football

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



Convention Highlights

The Ninth Annual Mu Alpha Theta National Convention was held August 5-8, 1979, at Athens State College, Athens, Alabama. Four hundred two participants from 21 states enjoyed the various sessions, activities, trips and competitions.

The general session speakers were Dr. Peter Casazza and Dr. F. Lee Cook of the University of Alabama in Huntsville, and Dr. John Neff of Georgia Tech. The section meetings included speakers on a variety of mathematical topics. Most of those attending the convention spent Tuesday evening at Point Mallard in Decatur and visited the Space & Rocket Center in Huntsville.

A majority of the students competed in at least one of the contests provided in mathematics, chess and stratego. The individual trophy winners were Alan Vonah of Wahlert High School, Dubuque, Iowa (first place in chess) and Brian Warwick of East Jefferson High School, Metairie, Louisiana (first place in stratego).

The top eight teams in the mathematics competition were 1) Austin High School, Decatur, Alabama; 2) New Trier East A, Winnetka, Illinois; 3) New Trier East B, Winnetka, Illinois; 4)Gainesville H.S., Gainesville, Florida; 5) Wentzville H.S., Wentzville, Missouri; 6) Walton H.S., Marietta, Georgia; 7) Wauwatosa West H.S., Wauwatosa, Wisconsin; 8) Christian Brothers H.S., Memphis, Tennessee.

The fifteen highest scoring individuals in the mathematics competition were 1) Richard Borie, Austin H.S.; 2) Michael Spertus, New Trier East H.S.; 3) Thomas Allen, New Trier East H.S.; 4) Kei-Mu Yi, New Trier East H.S.; 5) Eric Yuen, New Trier East H.S.; 6) William Watson, Gainesville, H.S.; 7) Andy Hallman, Austin H.S.; 8) John Rickert Wauwatosa West H.S.; 9) Doug Shors, Wentzville H.S.; 10) Tom Park, New Trier East H.S.; 11) Sam Hwang, Wentzville H.S.; 12) Chris Peterson, Gainesville H.S.; 13) Terry Chan, New Trier East H.S.; 14) Marvin Trapp, Jr., West Point H.S.; 15) Mike McCutchan, Wentzville H.S.

Convention co-chairmen Thomas N. Thrasher and Gwen Snoddy express their appreciation to all who participated in the convention. The 1980 convention will be held at Georgia Tech, Atlanta. See you then!

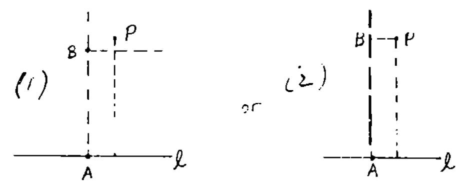
Reader Notes 'Non-fatal Error' In Short Ruler Article

Dear Math Log Editor:

There would appear to be an error in the otherwise very interesting article "Construction With A Short Ruler", in the Spring 1979 issue of the Math Log.

Specifically, given a short ruler 5 cm. long, as well as a compass with this same size restriction, assertion 3), found on page 4 of that issue, is not true.

To drop a perpendicular from a point P more than 5 cm. from line 1, do the following: Choose a point A on 1, preferably close to where the intended perpendicular will meet 1. Draw a perpendicular to line 1 through point A, and extend as long as necessary. Now choose a point B on the perpendicular which is reasonably close to P. Through B draw a perpendicular to the first perpendicular. If P is within 5 cm. of the new perpendicular, drop a perpendicular to it from P. Extend. This new line is the required perpendicular to it from P.



Note that the above construction appears to be predicated on having a "good eye". In fact, however, this is not so. If one, for example, chooses A too far off then one can discern this by his being unable to "reach" the first perpendicular with the 5 cm. compass having one end on P. It is then obvious how to rectify this by a new initial point A. In fact, one can then even simplify the above further, by next dropping a perpendicular from P to the first perpendicular and then another perpendicular to that through P, when extended, will yield the desired result. (See [2]).

Michael Ecker
Scranton,Pa.

Editor's Note: *Little and Dachtyl agree with Ecker while maintaining that this is not a "fatal flaw". If you see assertion 3 as a self-imposed restriction, you still have a valid argument and a an interesting situation.*

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Address correspndence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman OK 73019.

Variations On Magic Squares (continued)

Figure 7

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

This magic square is called *Melancolia* and was created by Albrecht Durer (1514). This is more than a magic square. If we add numbers of each row we get 34; each column also gives us 34; each diagonal adds to 34. Moreover, the center four numbers add to 34. There are other four numbers which add to 34. Can you find them? There are numbers in corners which add to 17. Can you find them?

It is easy to discover that many magic squares can be obtained from Figure 6 by adding a fixed number to every number of the chart. Try a few examples.

One also can obtain other magic squares from (Figure 6) by multiplying every element of this square by a fixed number. Thus, as in #1 one observes that with every 16 consecutive terms of an arithmetic progression one can construct a magic square. We leave the details to the reader.

3. **Magic Square of Order Five:** Here we shall write another magic square and leave the variations to the reader.

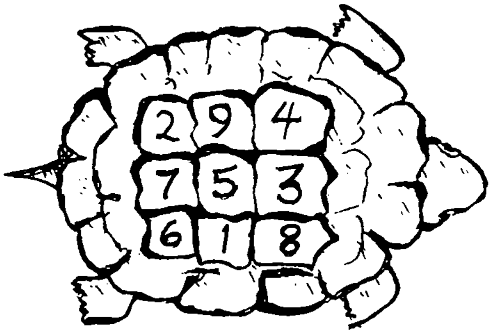
Here is a five by five magic square (Figure 8).

Figure 8

20	8	21	14	2
11	4	17	10	23
7	25	13	1	19
3	16	9	22	15
24	12	5	18	6

There are other 5 by 5 magic squares with integers one through 25. We shall only say that as soon as one obtains a magic square of order n, one has a general variation on it. This means that one can construct a magic square with n^2 consecutive elements of an arithmetic progression.

4. **A Note On The History:** In 2200 B.C. Chinese Emperor *Yu* has written a method of obtaining magic squares on the back of *Divine Tortoise.* Many



of the new ways of constructing magic squares were brought to Europe in the Middle Ages by travelers who had learned of them in Asia. For example, *De la Loubére,* a Frenchman, learned his method in Siam. He had been the Ambassador of Louis XIV in 1687-88. Long long ago, people believed that magic squares had mystic power. It was thought that a magic square engraved on a silver plate and worn around the neck would ward off the plague.

Other forms of magic squares have different names. Among them there is one called the Greco-Latin Square. These squares have many different uses. For example, controlled experiments in many fields may be made, including agriculture.

Reference:1 Startk, Harol M., *An Introduction to Number Theory*, Markham Publishing Co., Chicago (1970).

Editor's Note: Can you find a three-by-three magic square with all nine numbers prime? For more on this topic, see *Mathematics On Vacation* by Joseph S. Madachy.

U.S. Team Places Fifth In International Math Olympiad

The United States team competing in the 21st International Mathematical Olympiad placed fifth this summer in London, England. Teams from 22 nations competed and the USSR came in first.

Second place went to Rumania, followed by West Germany, England and the United States.

Last year the United States team finished second and they won the competition in 1977.

Team members were: Michael Larsen, Lexington, Mass.; Mark Pleszkoch, Manassas, Va.; Bruce Smith, San Rafael, Calif.; Michael Finn, Annandale, Virginia; Lawrence Penn, Great Neck,NY; Richard Agin, Chicago, Illinois; Randy Ekl, North Hunting-ton, Pa.; and Ronald Kaminsky, Albany, New York.

This team was chosen based on individual scores in the Annual High School Mathematics examination and the US Mathematical Olympiad. The United Sates will host the 1981 Olympiad.

News

Last spring Dr. Mike Keedy, professor at Purdue University, was keynote speaker at the installation services at Carver High School, Birmingham, Alabama. The chapter also held a lecture series throughout the year.

Florida's state convention will be held February 15-16 at Brandon High School, Brandon,FL.

The annual High School Mathematics Examination will be held March 4. Details are being mailed in November. A record number took the exam last year — 377764 — and Michael Finn of Annandale, VA had a perfect paper.