THE MATHEMATICAL LOG

Volume XXII, No. 3 April, 1978

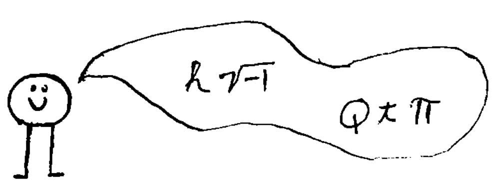
AN APPLICATION OF LOGARITHMS

One of the interesting facets of mathematics is that once any aspect of it is developed - - often in order to solve some specific problem - - it turns out to have all sorts of unanticipated uses in other areas. As a case in point, consider logarithms. Developed around 1600, the goal was to facilitate computation. The invention of logarithms was based on the realization that numbers in geometric progression ($a, ar, ar^2, ar^3,\ldots$) have exponents which are in arithmetic progression ($0, 1, 2, 3, \ldots$). In fact, the word "logarithm" was coined by John Napier by combining the Greek words "logos" for ratio and "arithma" for number - - literally "number of the ratio".

For over 300 years, the usefulness of logarithms to simplify computation was invaluable. Now, of course, we use electronic calculators and high-speed computers for serious computational work. It would be a mistake, however, to conclude that logarithms are obsolete, for they have turned out to be most useful in understanding many aspects of our world and ourselves. For example, physiologists have discovered that human beings perceive most stimuli in logarithmic fashion. The first scientific studies which led to this realization were performed by the German physiologist E.H. Weber, who found that most people can tell that a 21-gram weight is heavier than one weighing 20 grams. However, in comparing with a 40-gram object, usually 42 grams are needed before the heavier weight can be distinguished; in comparison with 60 grams, at least 63 grams and so on. Weber generalized his findings in 1846 with the following statement: "Noticeable differences in sensation occur when the increase in stimulus is a constant percentage of the stimulus itself." This result, usually referred to as Weber's "law", implies that when we perceive a sensation increasing in apparently equal steps (i.e., the stimuli appear to be in arithmetic progression), measurements show that the actual stimuli strengths closely approximate a geometric progression. To see that this is so, suppose that stimuli of intensities s_1, s_2, s_3, s_4 are sensed as increasing in equal steps. According to Weber, this means that $\frac{s_2-s_1}{s_1} = \frac{s_3-s_2}{s_2} = \frac{s_4-s_3}{s_3}$. But then $\frac{s_2}{s_1} - 1 = \frac{s_3}{s_2} - 1 = \frac{s_4}{s_3} - 1$, or $\frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3}$, and this is just the condition that makes s_2 the geometric mean between s_1 and s_3, and s_3 the geometric mean between s_2 and s_4.

It has been found that the perception of taste, smell, sound, and visual brightness is, for the most part, in accordance with Weber's principle; physiologists also feel that the response to drugs, for most organisms, is similar, although it is not always possible to measure such responses exactly.

If you have studied geometric sequences, you should be able to solve the following problems. Answers are provided at the end.



1. Our perception of musical pitch is in keeping with the principle enunciated by Weber. For example, if a certain note has the pitch whose frequency is 440 Hz, (Hz is the abbreviation for the unit Hertz; 1 Hz = 1 cycle per second) then the note one octave higher has frequency 880 Hz, and the note one octave lower has frequency 220 Hz. Under the current standards of piano tuning, the lowest C on the piano has frequency 33 Hz, and a piano keyboard has 8 C's. Find the frequency of all the C's on a properly tuned piano.

2. Suppose we want to test the human reaction to one of the hallucinogenic drugs. According to Weber's "law" the doses should form a geometric sequence if we expect equally spaced responses. (a) In such a series of tests, if the first two doses are 5 micrograms and 15 micrograms, what should the next three doses be? (b) If a series of four doses is planned, the smallest being 4 micrograms and the largest 500 micrograms, what should the other two doses be?

3. It has been suggested (N.O. Calloway, Science, 150, 1965) that experiments involving age in laboratory animals should use groups of animals whose ages form a geometric sequence. (a) If such an experiment is performed with first a group of mice 3 weeks old and then a second group 5 weeks old, what should the ages of the next two groups be? (b) If a characteristic of growth is studied in 5 groups of mice with the youngest 3 weeks old and the oldest 15 weeks old, what should be the ages of the three intermediate groups?

Some Logarithmic Scales for Perception

Now if we have a geometric sequence $a, ar, ar^2, ar^3,\ldots$, then the logarithms of the successive terms are, in order, $\log a, \log a + \log r, \log a + 2\log r, \log a + 3\log r,\ldots$, which forms an arithmetic sequence. This means that Weber's results imply that the human response to sensation is proportional to the logarithms of the stimuli. One consequence has been the development of a logarithmic scale for sound loudness. Interestingly enough, the scale for star brighness, whose basis was developed almost 2000 years before Weber lived, has also turned out to be logarithmic.

The decibel scale for sound loudness is defined by $B = 10 \log \frac{I}{I_o}$, where B is the sound level in decibels (db) of a sound of physical intensity I, and I_o is the physical intensity of a sound which can barely be heard by the average person. The following table gives the decibel level of some common sounds:

Decibels	Sound
0	barely audible
10	rustle of leaves
20	quiet whisper
30	quiet street (no traffic)
40	average home
50	quiet auto at a distance of 7 meters
60	normal conversation
70	noisy office
80	loud orchestra
90	"quiet" high school rock band
100	machine shop
110	pneumatic drill
120	pain level

continued on page 3

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics.
Address correspondence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman, Oklahoma 73019.

ANNOUNCEMENTS

The National Council of Teachers of Mathematics is considering a publication containing articles from the *Mathematics Teacher* that can be used with high school or two-year college mathematics students.

Anyone who has used the *Mathematics Teacher* as a resource in mathematics classes may already be aware of specific articles that are appropriate and well-received by students.

Suggestions regarding such articles to be included in the publication are welcomed. Please send bibliographical information to:

Dr. Betty Lichtenberg
Department of Mathematics
Education 307G
University of South Florida
Tampa, Florida 33620

1978 NATIONAL CONVENTION

Plan now to attend the 1978 convention, August 6-9, at Stevens Point, WI. Your chapter should have already received information on the meeting, but in case you have not, please write to Mr. Robert Meyer, Tomahawk High School, Tomahawk, WI 54487.

Over 200 chapters have expressed an interest, so it should be a great convention. Plan now to have a team in the Math Bowl competition, or some of your members may wish to present a student paper.

EDITOR'S NOTE: The amount of space available in *The Log* prohibited Dr. Wardlaw's complete analysis of the Game of Gogool (winter issue). He'll send you a copy if you write him---Department of Mathematics, U.S. Naval Academy, Annapolis, Maryland 21402.

CHAPTER NEWS

Two chapters recently chartered in Nova Scotia, Canada, and Geneve, Switzerland, indicate the universal appeal of mathematics.

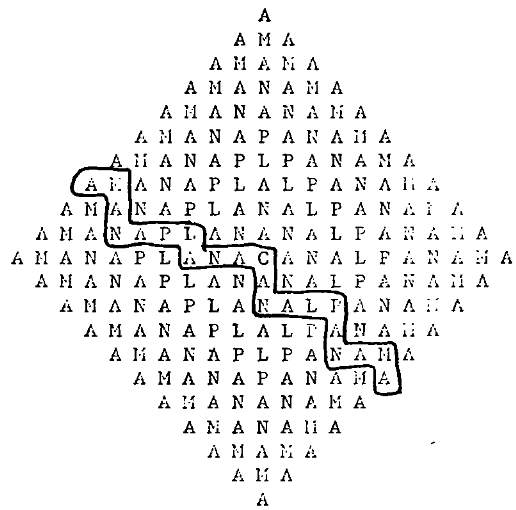
ABSOLUTELY |SILLY|

Branch of mathematics named after a bird. . . . . Owlgebra
(David Hoehl, Webb School of Knoxville)

Branch of mathematics named after a horse. .Triggernometry
(Author unknown?!?)

A MATHEMATICAL CANAL ANALYSIS...

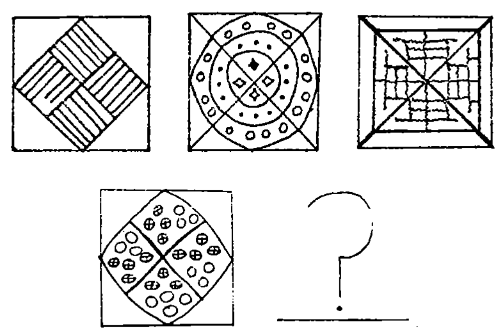
The expression A MAN, A PLAN, A CANAL, PANAMA is called a "palindrome"... Ignoring spacing, it reads the same from left to right as it does from right to left.



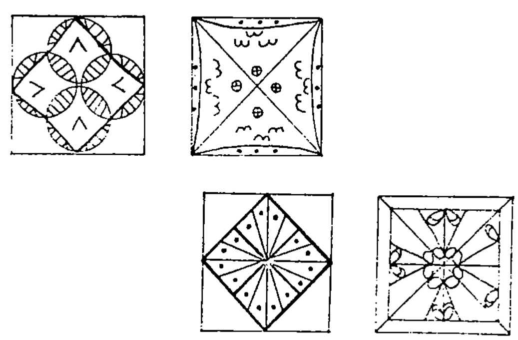
Consider the above arrangement. There are many ways to begin at an outside A, progress up, down, left, or right through the C, and end at an outside A, possibly the same one. One way is shown on the diagram. Would you believe that there are 16,744,464 ways to read "A MAN, A PLAN, A CANAL, PANAMA" in that diagram?

QUESTION: How do I know that? I surely didn't count them!

Study the first four designs to find a pattern.



Select one of the following designs to replace the question mark and continue the pattern:



by Andria Troutman
in 1974 FCTM Yearbook.

In response to "An Interesting Computation" (October 1977)
I can generalize somewhat—

— in base a—

$$\left(\tfrac{1}{2}a+n\right)^2-\left(\tfrac{1}{2}a-(n-1)\right)^2=a(2n-1)+(2n-1)$$

$$n\le\tfrac{1}{2}a$$

— ie. in base 10—

$n=1$, $2n-1=1$:

$$\left(\tfrac{1}{2}(10)+1\right)^2-\left(\tfrac{1}{2}(10)-(1-1)\right)^2=10(2(1)-1)+(2(1)-1)$$

$$6_{10}^2-5_{10}^2=10_{10}+1_{10}$$

$$36_{10}-25_{10}=11_{10}$$

$n=3$, $2n-1=5$:

$$\left(\tfrac{1}{2}(10)+3\right)^2-\left(\tfrac{1}{2}(10)-(3-1)\right)^2=10(2(3)-1)+(2(3)-1)$$

$$8_{10}^2-3_{10}^2=50_{10}+5_{10}$$

$$64_{10}-9_{10}=55_{10}$$

— pattern: $2n-1$ is nth odd integer
ie. $(1,3,5,7,9,11,\ldots,2n-1)$
$(1,2,3,4,5,6,\ldots,n)$

— for the next digit—

$$\left[a\left(\left(\tfrac{1}{2}a+n\right)-1\right)+\left(\tfrac{1}{2}a+n\right)\right]^2-\left[a\left(\left(\tfrac{1}{2}a-(n-1)\right)-1\right)+\left(\tfrac{1}{2}a-(n-1)\right)\right]^2=$$

$$(2n-1)(a^3+a^2+a+1)$$

$$a^3(2n-1)+a^2(2n-1)+a(2n-1)+(2n-1)$$

if $n\le\tfrac{1}{2}a$ 2 digits

if $n\le\tfrac{1}{2}a$ 4 digits

digit digit one's digit

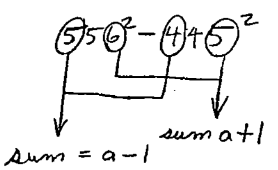
$$\left[10\left(\left(\tfrac{1}{2}(10)+2\right)-1\right)+\left(\tfrac{1}{2}(10)+2\right)\right]^2-\left[10\left(\left(\tfrac{1}{2}(10)-(2-1)\right)-1\right)+\left(\tfrac{1}{2}(10)-(2-1)\right)\right]^2=$$

$$67^2-34^2=1000(2(2)-1)+100(2(2)-1)+10(2(2)-1)+(2(2)-1)$$

$$4489-1156=3000+300+30+3$$

$$4489-1156=3333$$

all that remains is the challenged proof, however I shall leave that to someone better versed in the generalization proofs.



$$6+5 \qquad 10+1 \qquad 6=10-5+1$$

$$x+y=a+1 \qquad x=a-y+1$$

$$x^2-y^2=(a-y+1)^2-y^2=$$

$$\begin{array}{r} a-y+1\\ a-y+1\\ \hline a^2-ay+a\\ -ay\quad a+y^2-y+1\\ \hline a^2-2ay+2a-2y+y^2+1\end{array}$$

$$(a^2+y^2+1+2a-2ay-2y)-y^2=$$

$$a^2+1+2a-2y(a+1)$$

$$100+1+20-10(10+1)$$

$$121-110=11$$

anyway... (sigh!)

Brad Banko
Mansfield Malabar J.H.S.
Ohio 44907

continued from page 1

Unlike the decibel scale, which was proposed after Weber's results were known, the scale for star brightness has evolved from the magnitude scale developed by early Greek astronomers. They separated the visible stars into six categories, calling the brightest first magnitude, and the faintest sixth magnitude. Modern astronomical measurements have shown that there are appreciable differences in the brightnesses of stars originally assigned the same magnitude, but the ratio of the brightness of average stars in successive magnitudes is approximately constant. This is further evidence that stimuli which appear to us as equally spaced actually differ by a constant ratio.

The scale of star brightness in current use is based on the Greek ideas, but defined mathematically to express what is known about star brightness. Since a first magnitude star is about 100 times as bright as a sixth magnitude star, the magnitudes m and n of two stars of brighnesses b_m and b_n respectively are related by the equation $\frac{b_m}{b_n}=r^{n-m}$, where $r=\sqrt[5]{100}$. The more usual expression of this relationship is given by $n-m=2.5(\log b_m-\log b_n)$. The magnitude 1 has been chosen so that half the stars originally classified as first magnitude by the Greeks are brighter and half are fainter than a star of magnitude 1 would be.

Using this scale, the brightest star in our sky, Sirius, has magnitude −1.6, the star closest to the solar system, α Centauri has magnitude 0.1, the planet Venus at its brightest has magnitude −4, and the sun has magnitude about −27.

If you have studied logarithms, you may want to try the following problems:

4. Show that the equation $n-m=2.5(\log b_m-\log b_n)$ follows from the relation $\frac{b_m}{b_n}=r^{n-m}$, where $r=\sqrt[5]{100}$.

5. Show that if two sounds of intensities I_1 and I_2 have decibel levels B_1 and B_2 respectively, then $B_1-B_2=10\log\frac{I_1}{I_2}$.

6. From the decibel scale above, how much bigger is (a) the sound intensity of normal conversation than that of a whisper? (b) the sound intensity of a machine shop than that of a quiet street?

7. Two tones of the same frequency register 45 db and 60 db respectively on a decibel meter. What is the ratio of their actual physical intensities?

8. If one sound has an actual physical intensity 500 times that of another sound, what is the difference in the decibel levels of these sounds?

9. The star Aldebaran has magnitude 1.0; show that Sirius is about ten times as bright as Aldebaran.

10. When Venus has magnitude −4, how much brighter is it than a sixth magnitude star?

We have shown one of the ways that logarithms have been found useful; there are many others. This is characteristic of mathematical concepts. Generally developed to solve some specific kind of problem, they prove to be useful in other unanticipated circumstances. No doubt this is part of the importance of mathematics to society.

Answers to the problems:

1. 33,66,132,264,528,1056,2112,4224(Hz) 2. (a) 45,135,405 (in micrograms) (b) 20,100 (in micrograms) 3. (a) 8+(1/3) weeks, 13+(8/9) weeks (b) 4+(1/2) weeks, 6+(3/4) weeks, 10+(1/8) weeks. 6. (a) 10,000 times as big (b) 10,000,000 times as big 7. 1:31.6 8. about 27 10. 10,000 times as bright

Bernice Kastner
Montgomery College
Rockville, Maryland

WOMEN AND MATHEMATICS---THEN

Amalie Emmy Noether

The algebraist, Amalie Noether, was born on March 23, 1882, in Erlangen, Germany. She was the eldest child of Max Noether and Ida Amalia Kaufman who had three other sons. Her father was a mathematics professor of stature at the University of Erlangen. Both Emmy (as she was called) and her younger brother Fritz followed in their father's footsteps.

Emmy first attended the State Girls's School in Erlangen where she studied arithmetic, languages, and music. In 1900, she was certified as a teacher of English and French. She decided to continue her education at the university which proved to be difficult since there were all sorts of restrictions against women students. From 1900 to 1902 she audited mathematics and foreign language classes at Erlangen. During the following academic year, she audited mathematics classes at the University of Göttingen. In 1904, she returned to the University of Erlangen and was allowed to matriculate officially in the mathematics program. She completed a doctoral dissertation in algebra under the tutelage of Gordan and received the Ph.D. in 1907, summa cum laude. Her doctoral dissertation was entitled "Uber die Bildung des Formensystems der ternären biquadratischen Form". In the years that followed, she helped her father. Her own research also progressed influenced by the works of two other algebraists--Ernest Fischer and Erhard Schmidt.

Following her father's retirement and her mother's death, Emmy moved to Göttingen in 1915. There she became a contributing member of the research group centered around Hilbert and Felix Klein. For many years, she received no formal appointment as lecturer at Göttingen since it was against university policy to grant such positions to women. Finally, in 1922, Emmy was named "nichtbeamteter ausserordentlicher Professor" (unofficial associate professor), a title with no salary. She was given a "Lehrauftrag" (lectureship) in algebra in 1923 which brought her a small salary.

In 1920, she co-authored a paper with Schmeidler on differential operators which showed her strong interest in the conceptual axiomatic approach. There followed over a decade of research work culminating in various published papers co-authored with such algebraists as H. Hasse and Richard Brauer.

Emmy proved to be an effective, innovative teacher as well as an exceptional researcher. Prejudice and tradition, however, made it difficult for her to get a better position at the university. Her reputation spread and she was asked to deliver a series of lectures at Moscow University during the 1928-29 academic year. She also lectured at Frankfurt in the summer of 1930.

This period of productive research and teaching came to an abrupt end with the ascent of the Nazis to power. Her stature as a mathematician could not outweigh her femininity, her Jewishness, or her liberalism. She was dismissed from her position and forced to flee. Here, in America, she found a new home at Bryn Mawr where she served as visiting professor. She also lectured weekly at the Institute for Advanced Study in Princeton. Unfortunately, her newly found happiness was short-lived. She died suddenly on April 14, 1935, at the age of fifty-three leaving a legacy of over thirty research articles. What cannot be measured precisely is the vast extent of her influence on the development of abstract algebra through the interchange of ideas with students and fellow colleagues.

Louise S. Grinstein
Kingsborough Community College
of the City University of New York
Brooklyn, New York

WOMEN AND MATHEMATICS---NOW!

WAM - Women and Mathematics, a lectureship program designed to encourage young women in tenth grade to study more mathematics, reached well over 12,000 students and 1,100 teachers, parents, and guidance counselors during its second year of operation in 1976-77. Through visits to more than 115 high schools in New York/New Jersey/Connecticut, San Francisco Bay, and the Greater Chicago area, WAM speakers brought the message that sound preparation in mathematics is a necessary prerequisite for entrance into virtually every major career field and that women - heretofore frequently discouraged from taking more than just the required math - can be math achievers. The speakers themselves serve as role models for the students since they know and describe the value of math in their own careers which include ophthalmology, accounting, computer science, statistics, medicine, physics, urban planning, college teaching, research biology, engineering, and business management.

Since its inception in 1975, WAM has been sponsored by the Mathematical Association of America (MAA) under a grant from IBM. The idea for WAM was sparked when IBM representatives observed that no young women have been among the winners of the U.S.A. Mathematics Olympiad. The U.S.A. Olympiad is the math contest for certain invited high school students who have scored well on the "Annual High School Mathematics Examination," sponsored nationally by the MAA, National Council of Teachers of Mathematics, (NCTM), Mu Alpha Theta, and the Casualty Actuarial Society. A special team is chosen from among the U.S.A. Olympiad winners to compete in the International Olympiad held each summer. For the first time the U.S.A. team won top honors at the 1977 Olympiad. (See the October, 1977, issue of the Log.)

Evaluations of WAM by those who participate in its visits indicate that WAM has been achieving its goals. Demand for visits is steadily increasing, and the program looks ahead to presentations to high schools as well as to associations of guidance counselors, parents, and teachers, state officers of education, college audiences of elementary and secondary education majors, and others who influence the course and career choices of young women.

The national WAM program is directed by Dr. Eileen L. Poiani of Saint Peter's College, Jersey City, New Jersey. Dr. Poiani is pleased to announce that Johnson & Johnson has joined IBM in providing corporate support for the 1977-78 program. Such support will enable WAM to operate four regions; the regions and their respective coordinators are as follows:

New York/New Jersey
Lorraine Denby & Susan Devlin
Bell Laboratories
600 Mountain Avenue
Murray Hill, NJ 07974

Connecticut/Westchester, NY
Ms. Margaret Pryor
39 Robert Lane
Wappinger Falls, NY 12590

San Francisco Bay
Prof. Jean J. Pedersen
Dept. of Mathematics
University of Santa Clara
Santa Clara, CA 95053

Chicago
Dr. Bernadette Perham
4160 W. Eddy Street
Chicago, IL 60641

A list of speakers in your area and their topics, as well as application forms for participation in this program, are available from the appropriate coordinator.

Eileen Poiani
St. Peter's College
Jersey City, New Jersey