THE MATHEMATICAL LOG

Volume XX, No. 3 May, 1976

ISOMORPHISMS AND COMPUTERS

It is the impression of many people that: "Logarithms for calculation purposes are much horse and buggy stuff with the hand held calculators so common now." This article does not intend to use logarithms for calculation purposes. In fact, we suggest the use of computers for obtaining logarithms. Since the process involves interesting and simple ideas of mathematics, it is worthwhile studying.

1. An Isomorphism: Let us consider the sets N = {..., .01, .1, 1, 10, 100, ...} and L = {..., -2, -1, 0, 1, 2, ...}. For N we choose multiplication as its binary operation and for L we choose the addition. One observes that N can be written as powers of 10, i.e.,

$$N = \{\ldots, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, \ldots\}.$$

Thus N is closed under multiplication which means the product of any two elements of N is also an element of N. Similarly, we observe that L is closed under addition.

One can establish a one-to-one correspondence between N and L which preserves binary operations; namely, we define a function F of N onto L as:

$$F(10^n) = n,\ F^{-1}(n) = 10^n.$$

By the properties of exponents we see that

$$F([10^n][10^m]) = F(10^{m+n}) = m+n$$

and

$$F^{-1}(m+n) = 10^{m+n} = [10^m][10^n].$$

If one has already studied common logarithm, one recognizes that F is $\log_{10}$ and F^{-1} is the antilog.

Indeed, it is proper to ask why one wants to complicate the definition of common logarithm? This will be studied in what follows.

2. Inserting means: Note that N is a geometric progression with ratio 10 and L is an arithmetic progression with difference 1. We know that there are more positive numbers than the integral powers of ten, and we are interested in obtaining their logarithms. Thus we start inserting means. In N a geometric mean will correspond to an arithmetic mean in L. We know that the geometric mean of a and b is $\sqrt{ab}$ and the arithmetic mean of p and q is $\frac{1}{2}(p+q)$. For example, let 10^n, 10^m be members of N, then

$$\sqrt{[10^n][10^m]} = 10^{\frac{m+n}{2}}$$

This shows that

$$F\left(\sqrt{[10^n][10^m]}\right) = \tfrac{1}{2}(n+m)$$

which means the geometric mean of two elements of N corresponds to the arithmetic mean of the corresponding elements in L and vice-versa.

3. Computing logarithms: We shall give an example of how a table of common logarithm can be constructed. We know $\log 1 = 0$ and $\log 10 = 1$. The geometric mean of 1 and 10 is $\sqrt{10}$, and the arithmetic mean of 0 and 1 is $\frac{1}{2}$. Therefore $\log \sqrt{10} = \frac{1}{2}$. But to put this value in the table, we have to get an approximation of $\sqrt{10}$. We shall compute this up to three decimal places, i.e., 3.162. Thus, we say log 3.162 is approximately .5. Now we continue. The geometric mean of 1 and 3.162 is $\sqrt{3.162}$ which is approximately 1.778 and the arithmetic mean of 0 and .5 is .25. Thus an estimate of the log 1.778 is 0.25. We shall enlarge N by inserting geometric means, by multiplication and division of new and old elements. We also enlarge L by corresponding operations, i.e., inserting arithmetic means, by addition and subtraction. Gradually a log table is obtained. For example, the product (3.162)(1.778) approximately corresponds to sum .5 + .25 = .75.

4. Suggestions: Students usually look square roots in a table. This article is written considering no table is available. Thus one should learn Hindu method of extracting square roots. One may use other methods of approximating $\sqrt{a}$ for a real number a but the Hindu method is more accurate. (1)

Ideas of this article may be assigned to students as early as they can grasp the concepts. Some of computing may be done by machines.

Ali R. Amir-Moéz
Mathematics Department
Texas Technological Univ.
Lubbock, TX

(1) Ali R. Amir-Moéz, James N. Javaher, Precalculus Mathematics: Edwards Brothers Inc., Ann Arbor, MI 48104. pp. 48-51.

1976 NATIONAL CONVENTION

IT'S NOT TOO LATE TO REGISTER FOR THE NATIONAL CONVENTION!

THE PROGRAM LOOKS GREAT!!

EXCITING COMPETITION---EXCELLENT COMMENTATORS---

EXHILARATING COMPANY !!

Date: August 9-11, 1976
Location: West Chester State College
West Chester, Pennsylvania
Contact: Mr. Philip Connor
730 Gay Street
Phoenixville, PA 19460

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. Address correspondence to: Mu Alpha Theta, 601 Elm Avenue, Room 423, The University of Oklahoma, Norman, Oklahoma 73069.

SECRETARY'S CORNER

New members as of May 1, 1976, total 14,926 which is approximately 1,880 more than May 1, 1975. Keep up the good work.

Since the last issue of The Mathematical Log, state and regional meetings have been reported to our office as follows: (1) Texas State Convention, February 6-7, 1976, Stephen F. Austin High School Chapter, Audrey Nugent-sponsor; (2) Mississippi State Convention, March 6, 1976, Murrah High School Chapter, Pauline Tramel-sponsor; (3) Tennessee State Convention, March 11-12, 1976, Messick High School Chapter, Mary Truebger-sponsor; (4) Alabama State Convention, April 23-24, 1976, Robert E. Lee High School Chapter, Susan Barmby-sponsor. We understand that Louisiana had their usual big annual state meeting but have not received their report.

Local and regional meetings were as follows: (1) Pecos, Texas chapter, January 17, 1976, Carolyn Rankin-sponsor; (2) Andrews, Texas chapter, January 31, 1976, Ken Williamson-sponsor; (3) Miami Beach, Florida chapter, March 31, 1976, Milton Zoloth-sponsor; (4) San Antonio, Texas chapter, April 29, 1976, Paul Foerster-sponsor; (5) Goose Creek, South Carolina chapter, May 8, 1976, Ann Long and Cheryl Coker-sponsors.

Our thanks to all of the clubs and their sponsors for hosting these meetings. Our apologies if we missed reporting any meeting.

Florida is having its First Annual State Convention this month. The host club is the Columbia High School chapter of Lake City under the sponsorship of Ms. Patricia Casey.

Once again the Pecos High School chapter of Pecos, Texas, is sponsoring a Texas Regional Conference meeting. (January 15, 1977) Contact Brian Gross for more information.

ANNOUNCEMENTS

From an article written by Dr. Frank Anderson, Chairman of the Dept. of Physical Engineering at the University of Mississippi.

> "In recognition of the important role in secondary education played by the Mississippi chapter of Mu Alpha Theta, nationwide honorary mathematics society, the School of Engineering and the Dept. of Mathematics at the University of Mississippi are pleased to sponsor a special Mu Alpha Theta SPEAKERS SERVICE. All Mu Alpha Theta clubs in Mississippi are invited to use the SPEAKERS SERVICE on a no-cost basis."

A list of speakers and topics was included as well as complete instructions for requesting a speaker.

This is an excellent example of cooperation and support. If you are in Mississippi, don't hesitate to take advantage of this tremendous offer. If you are in another state, start now to enlist support such as this from one of your universities.

MATH FAIR STUDENT PAPERS

Twenty-two papers are now being read for possible inclusion in the first booklet. This is a continuing project so plan now to work on a paper for next year. For more information look in the last two issues of The Mathematical Log or contact Harry Ruderman, Hunter College High School, 466 Lexington Avenue, New York, NY 10017.

REVISION OF SPONSOR'S HANDBOOK

Dr. Margaret Maxfield, sponsor of the Manhattan High School chapter has agreed to work on a revision of The Handbook. She would like material on: (a) Initiation ceremonies; (b) How to operate a local chapter, that is, program ideas and other activities. Send the material to our office or to Dr. Margaret Maxfield 417 N. 17th Street, Manhattan, KS 66502.

We regret to announce the recent death of Margaret Dam, sponsor of Oakland High School chapter in Murfreesboro, Tennessee. Mrs. Margaret Dam was a sponsor at Oakland High School since 1965.

ANSWER TO LAST CRYPTARITHM

MOON		9552	E = 0
MEN		902	R = 1
CAN	SOLUTION	382	N = 2
REACH		10836	C = 3
			O = 5
			H = 6
			A = 8
			M = 9

ACUTE WELCOMING SPEECH

I'm not very well Coordinate - ted, but I want to make One Point Graphically Plane. Our basic Assumption is that Mu Alpha Theta is a very Unique Set. We are glad that your Operations have Formula - ted a Union in Ten-essee, and - Brace yourself - that you have Intersected here on our campus today. We Complement you. I thought about saying we are glad you are meeting here in Disjoint, but I was afraid the Dean might be listening. I will say that we are delighted to have you Range around in our Domain. Nothing could be Greater Than our joy at having you in our Neighborhood for your First Annual Function. In fact, the Implication is that our joy knows no Limits. Yesterday you had a very Finite, and we are Determinant that today's activities will be Positive Additions to that, with our speakers being the Prime Factors. By now you have realized that all of us here in this Auditorium are Real people - you are all Rational, and I, of course, am Irrational. It may sound Odd, but Even though I don't know the Identity of each of you, you do Exist, and you are Unique, and I am glad to have this Chance to Converse with you, and to become a Closer Associative with the distinguished members of Mu Alpha Theta. If you should come up to me sometime today and ask me, "Is Mu Alpha Theta welcome at Memphis State University?", I would say, "Are EUCLIDEAN?"

by Dr. J.F. Crabtree, Memphis State University at Tennessee Mu Alpha Theta State Convention, March, 1976.

ABSOLUTELY |SILLY|

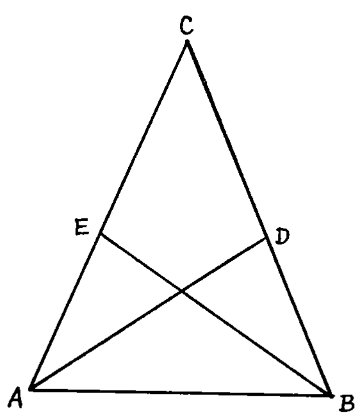
What do you tell Al when his parrot dies?....polygonal
(Rick Wyatt)

A boring radish is a....square root
(Kathy Warren)

What does a magician perform in the month following April?.....matrix
(Greg Dick)

A champion quacker is a....product
(Debbie Fasick)

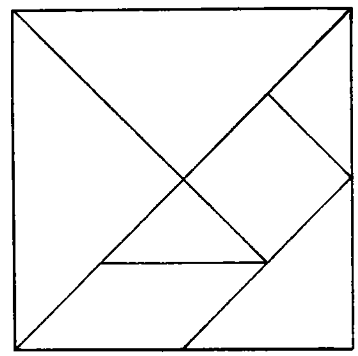
NOT SO SILLY



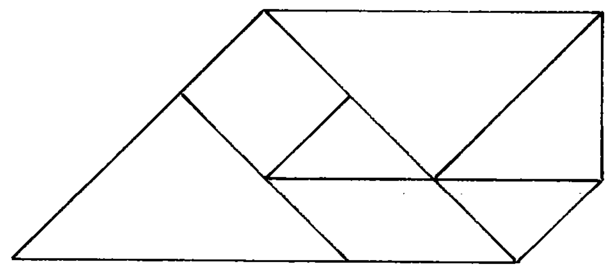
In the figure, suppose that $\overrightarrow{AD}$ bisects $\angle BAC$, $\overrightarrow{BE}$ bisects $\angle ABC$, and AD = BE. Can you prove that $\triangle ABC$ IS isosceles? (The converse of this is a lot easier to prove.)

TANGRAMS

You may be familiar with the ancient Chinese TANGRAM puzzle. It consists of seven pieces that can be cut from a square piece of poster board like this.



A square is an example of a convex polygon. The Tangram pieces can be arranged to form twelve other convex polygons. Here is another example.



See if you can form the other eleven convex polygons. (You must use all seven pieces each time.)

CRYPTARITHM

A Cryptarithm is a puzzle made by substituting letters for digits in a simple arithmetic problem.

A given letter stands for the same digit throughout the puzzle. No digit is represented by more than one letter.

```
   TWO
 THREE
 SEVEN
------
TWELVE
```

Answer next issue

Experimental units on teaching cryptarithms may be obtained for $4.50 from Dr. and Mrs. Andree, Mathematics Department, University of Oklahoma, Norman, Oklahoma 73069, for classroom trial. (10 or more copies $2.90 each)

AN EASIER WAY TO CONVERT? Kurt Bowman

Kurt Bowman, a senior at Southeast High School in Bradenton, Florida, submitted this procedure for converting from another base to a base ten numeral. The question for you is, "WHY DOES THIS WORK?"

Example: $21425_{(six)}$

Problem: Convert to base 10

A. List given base 6 numerals in a column, in order, first number at bottom, through last number at top. See Diagram 1.

DIAGRAM 1.

```
2  1  4  2  5 (six)
               → 5
               → 2
               → 4
               → 1
               → 2
```

B. Multiply bottom number by given base and add next ascending digit. See Diagram 2.

DIAGRAM 2.

```
      5
      2
      4
→ 12 + 1 = 13
  (6)  2 = 12
```

C. Take answer from Step B (Diagram 2), multiply by given base, and add next ascending digit. See Diagram 3.

DIAGRAM 3.

```
      5
      2
→ 78 + 4 = 82
→ 12 + 1 = 13, 13·6 = 78 →
  (6)  2 = 12 →
```

D. Take answer from Step C (Diagram 3), multiply by given base, and add next ascending digit. See Diagram 4.

DIAGRAM 4.

```
→ 492 + 2 = 494, 494 · 6 = 2964
→  78 + 4 =  82,  82 · 6 =  492 →
→  12 + 1 =  13,  13 · 6 =   78 →
    2 (6) =  12 →
```

E. Repeat procedure until all ascending digits have been used. See Diagram 5.

DIAGRAM 5.

```
→ 2964 + 5 = 2969, the answer
→  492 + 2 =  494, 494·6 = 2964 →
→   78 + 4 =   82,  82·6 =  492 →
→   12 + 1 =   13,  13·6 =   78 →
     2 (6) =   12 →
```

F. The resulting number is your given number converted to base 10. See Diagram 5.

IT'S GREATER THAN I THOUGHT

Brian Galuska, Jeff Glass, Richard Vickroy, and Frank Smigla of Bishop McCort High School in Johnstown, PA., have called my attention to an error in my article, "It's Greater Than You Think" in the last issue of The Mathematical Log. I made the statement that:

$$2^{50} = 1,125,602,549,381,344.$$

The four Bishop McCort students correctly pointed out that the statement should be:

$$2^{50} = 1,125,899,906,842,624.$$

I hate to admit that I made a mistake, but this at least gives me a chance to discuss a method for carrying out such computations on a mini or pocket calculator. You may find the method useful in your school work.

If a minicalculator has an eight-digit display (as most do), it is possible to compute 2^{25}. You will find that

$$2^{25} = 33,554,432.$$

Now to compute 2^{50} we make use of the fact that $2^{50} = (2^{25})^2$ and also the fact that $(a+b)^2 = a^2 + 2ab + b^2$ for all real numbers a and b. The reasoning is as follows:

$$\begin{aligned} 2^{50} = (2^{25})^2 &= (33554432)^2 \\ &= (33550000 + 4432)^2 \\ &= (3355 \times 10^4 + 4432)^2 \\ &= (3355 \times 10^4)^2 + 2(3355 \times 10^4)(4432) + 4432^2 \\ &= (3355^2 \times 10^8) + (2 \times 3355 \times 4432 \times 10^4) + 4432^2 \end{aligned}$$

Now the minicalculator can be used to determine that:

$3355^2 = 11256025$,
$2 \times 3355 \times 4432 = 29738720$,
and $4432^2 = 19642624$.

Hence, 2^{50} is the sum of these three numbers:

```
1125602500000000
     297387200000
         19642624.
```

It is easy to compute this sum without a calculator and arrive at the conclusion that:

$$2^{50} = 1,125,899,906,842,624.$$

This is the method that I used previously, but I made a careless mistake. Can you figure out where?

Actually, I was not too far from the correct number. I was only off by 297,357,461,280, and that is an error of less that .03 of 1%!

D. R. Lichtenberg