THE MATHEMATICAL LOG

Volume XIX, No. 1 September, 1974

FIBONACCI NUMBERS IN NATURE

FIBONACCI SEQUENCES

The Fibonacci sequence is as follows: $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$, $F_8 = 21$, $F_9 = 34$, $F_{10} = 55$, $F_{11} = 89$, $F_{12} = 144$, $F_{13} = 233$,........ The sequence builds up by adding the two previous terms to get the next term. Any sequence that develops in this way is known as a Fibonacci sequence. Thus: 2,5,7,12,19,31,50.......

Another special Fibonacci sequence which is found more or less commonly in nature is the Lucas sequence which starts with the numbers 1,3. The Lucas sequence is as follows: $L_1 = 1$, $L_2 = 3$, $L_3 = 4$, $L_4 = 7$, $L_5 = 11$, $L_6 = 18$, $L_7 = 29$, $L_8 = 47$, $L_9 = 76$, $L_{10} = 123$, $L_{11} = 199$, $L_{12} = 322$, $L_{13} = 521$........

LEAF ARRANGEMENT

The arrangement of leaves in plants is known by the scientific name, phyllotaxis. From the standpoint of Fibonacci numbers, two things are observed: (1) The number of leaves it takes to go from any given leaf to another leaf similarly placed on the stem; (2) The number of turns as one follows the leaves in going from one leaf to the next leaf similarly placed. Both these numbers turn out to be Fibonacci numbers in many instances.

In the case of leaf arrangement, the following type of notation is used: 3/8 means that it takes three turns and eight leaves to arrive at the next leaf in corresponding position. Another way of thinking of this is as follows: Since it takes eight leaves to cover three revolutions, the average angle gone through in passing from one leaf to another is 3/8 of a revolution or 3/8 of 360 degrees.

The following table shows the angle of turning from one leaf to the next for various types of leaf phyllotaxis.

PHYLLOTAXIS TYPE	CALCULATION OF ANGLE	ANGLE
1/2	(1/2) 360^o	180^o
1/3	(1/3) 360^o	120^o
2/5	(2/5) 360^o	144^o
3/8	(3/8) 360^o	135^o
5/13	(5/13) 360^o	138.5^o
8/21	(8/21) 360^o	137.1^o
13/34	(13/34) 360^o	137.6^o

Note that in giving the phyllotaxis type alternate Fibonacci numbers are used.

In regard to leaf arrangement, it should be pointed out that we are talking about a statistical average. In other words, you may examine plants and find that while the majority of them have a given Fibonacci pattern, in some you may be unable to find it while in others there is a variant Fibonacci or Lucas pattern. What we are talking about is evidently an approximation from the very fact that the precise location of a leaf or bud is not a point!

In plants that have a long stem, it may well be that one type of phyllotaxis can be noted near the bottom of the plant while as we proceed up the stem, the type changes. Thus in one desert agave, for the first five there was a count of 2/5 to the left, while for the positions 6 to 18, the count was 3/8 to the left.

In the OCOTILLO, there are growth segments and one should count within each segment.

With these cautions in mind, the following list is given for the phyllotaxis of some common plants.

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ANNUAL HIGH SCHOOL MATH CONTEST

The date for the Annual High School Mathematics Contest, sponsored by the NCTM and other organizations, has been set for March 11, 1975. Registration, which is handled regionally, closes January 15, 1975. A list of regional coordinators is included with this mailing of the Log. If you have any questions, please contact your regional coordinator.

HYPERBOLIC GEOMETRY

Hyperbolic geometry is a relative newcomer on the mathematical scene. As early as 3000 B.C. the Egyptians understood and used the ratio pi and the properties of the 3,4,5 triangle. It has been conjectured that Egyptian mathematics was embodied primarily in engineering considerations. Aside from such practical approaches, a systematic development of geometry as we know it today had to wait until approximately 300 B.C., when Euclid wrote his treatise, The Elements. This was the first comprehensive, logical system of geometry.

Euclid's basis for The Elements was five "common notions" and five "postulates", which are referred to collectively as the ten "axioms". These ten were assumed to be true, and from them Euclid developed his theorems through deductive proof. The first nine axioms are relatively easy to understand, because they are localized in their effect. However, the fifth postulate is not so self-evident; the fact that it deals with lines extending to infinity makes it difficult to visualize.

The fifth or "parallel" postulate, as it is commonly called, is paraphrased in Playfair's Axiom, "there is one and only one line parallel to a given line through a given point not on the given line". Playfair's Axiom differs from the fifth postulate as given in The Elements. It is one of a number of equivalent statements such as "space can be subdivided into equal cubes", or "the angle sum of a triangle equals 180°".

Euclid perhaps anticipated difficulties concerning the fifth postulate, and proved the first 28 theorems using only the first nine axioms. These form the basis of "absolute" geometry since its theorems do not depend on the parallel postulate. Through the centuries following Euclid, many mathematicians attempted unsuccessfully to deduce Euclid's fifth postulate from the other nine axioms.

One of the most famous such attempts was by a seventeenth century Jesuit, Sacheri. By means of a figure now known as the Sacheri Quadrilateral (fig. 1) he tried to prove the fifth postulate. Sacheri found that he could only prove the summit angles A and B congruent, and not right as he had hoped. He was faced with three possibilities; that the angles were right, acute, or obtuse. These assumptions lead, respectively, to Euclidean, Lobachevskian, and Riemannian geometries. These hypotheses are also equivalent to assuming 1, at least 2, or no parallels to a given line, through a given point.

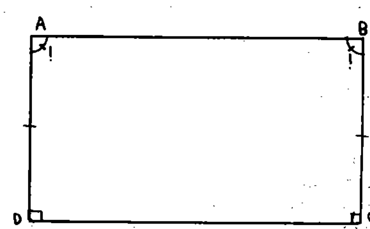

Fig. 1

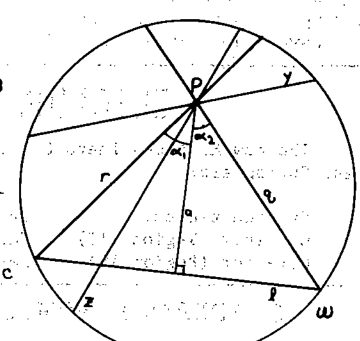

Fig. 2

Gauss was one of the first to observe the independence of the fifth postulate, but Bolyai and Nikolai Lobachevski were the first to publish a logical development of a geometry based upon a non-Euclidean substitute. They chose to substitute an assumption of two critical parallels, and independently derived a large number of surprising theorems. Beltrami showed the new logical structure was free from contradictions, and Klein suggested the name "hyperbolic" geometry.

One possible model of the hyperbolic plane is the Klein model. The entire hyperbolic plane lies within a Euclidean circle, designated ω (fig. 2). The circle itself represents a set of ideal points which do not lie within the hyperbolic plane, but which serve as limits. Hyperbolic lines (h-lines)

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THE MATHEMATICAL LOG

September, 1974 Volume XIX, No. 1

The official publication of the National High School and Junior College Mathematics Club, Mu Alpha Theta, which is sponsored by the Mathematical Association of America and National Council of Teachers of Mathematics. Address correspondence to Mu Alpha Theta, Department of Mathematics, 601 Elm, Room 423, The University of Oklahoma, Norman, Oklahoma 73069.

SECRETARY'S CORNER

Last year was a good year for Mu Alpha Theta. Some of the highlights were: (a) 15,834 new members joined Mu Alpha Theta, (b) 73 new clubs were chartered, (c) a copy of our joint publication with NCTM, "Topics for Math Clubs", was sent to each chapter, (d) state organizations are now flourishing in Louisiana, Mississippi, Alabama, Texas, Tennessee and Kentucky, (e) a very successful national convention was held on August 4-7, 1974, at the University of Arkansas (Approximately 400 students from coast to coast enjoyed three days of talks on mathematics, math bowl competition and just getting acquainted with each other.),(f) saving the good news for the last, we are solvent with $25,680 in our treasury as of June 30, 1974.

For next year we are planning a reprinting of the Handbook for Sponsors and would appreciate suggestions on topics that need to be added, topics that should be improved and any that might be deleted.

We are still co-sponsoring the Annual High School Mathematics Contest and will be sending a list of Regional Coordinators for 1974-75 with one of the mailings of the Log.

If any state group wishes information on how to get started, we have some information in our office which would be helpful, so please write.

Of course, the big event will be the sixth annual convention scheduled for the first week of August in Seattle, Washington. More information will be sent on this later, but start making plans now to attend.

Have a good Mu Alpha Theta year.

ELECTION RESULTS

The new Mu Alpha Theta Governing Council members elected last Spring are:

President-Elect	Sarah T. Herriot
Governor (Region III)	Harry Ruderman
Governor (Region IV)	Betty Lichtenberg

FIBONACCI NUMBERS (continued)

Toyon	2/5
Coast live oak	2/5
Madrone	2/5
California bay	2/5
Bottle brush	5/13 in both the leaves and the flower outlets
Mustard	2/5
Black acacia	2/5 and 3/8 arrangements found
Fiddleneck	1/3
Shepherd's purse	2/5 in the fruits
Pepper tree	2/5
Lombardy poplar	2/5 in the buds (later leaves)
Common groundsel	2/5
Petty spurge	3/8
Willow	2/5 or 3/8. Quite variable.
Wild blackberry	1/3 or 2/5
Holly	2/5
Manzanita	2/5
Apple	2/5

RATE YOUR WITS

Without using tables, slide rules, calculators, computers, calculus, or anything but your own wits, determine which of the following quantities is greater.

$$\sqrt{e^{\pi}} + \sqrt{\pi^{e}} \text{ or } \sqrt{e^{\pi} + \pi^{e}}$$

FIBONACCI ARRANGEMENTS IN PINE CONES

The bracts on a pine cone are considered to be modified leaves which have been compressed into a smaller space. However, due to this compression it is more convenient to think of the Fibonacci pattern in other terms. What is obvious to observation is two sets of spirals, one to the left and the other to the right, one steep and the other more gradual. Sometimes the steep spiral goes to the left, sometimes to the right as we hold the cone with its point downward.

Two principal systems of an equivalent nature may be used in making the count. The first is to consider the steep spirals and find how many there are: **this should be a Fibonacci number. Then** look at the gradual spirals and count their number; this should be an adjacent Fibonacci number.

The other method is to take any bract near the top of the cone and go down along the two spirals passing through the bract until one comes to the intersection of the spirals. Then count down along the spirals to the intersection. There will be two adjacent Fibonacci numbers, the larger being the count along the more gradual spiral. We can establish the following convention for indicating the result.

(1) Hold the cone with its point downward. Choose any bract.
(2) Count to the next intersection along the gradual spiral. If there are eight steps to the left, call this 8L.
(3) Count to the next intersection along the steep spiral. This should be 5R.
(4) This count would be denoted 8L-5R.

In addition to the two obvious spirals, it is possible to establish other spirals. For each pair of spirals, there will be a pair of Fibonacci numbers going with them. The following table gives results for California pine species.

Pinus albicaulis (Whitebark Pine)..5-3,8-3,8-5
Pinus flexilis (Limber pine)..8-5, 5-3, 8-3
Pinus Lambertiana (Sugar pine)..8-5, 13-5, 13-8, 3-5, 3-8, 3-13, 3-21
Pinus monticola (Western white pine, Silver pine)..3-5
Pinus monophylla (One-leaved pinyon).. 3-5,3-8
Pinus edulis..5-3
Pinus quadrifolia (Four-leaved pinyon).. 5-3
Pinus aristata (Bristlecone pine)..8-5, 5-3, 8-3
Pinus Balfouriana (Foxtail pine)..5-8, 5-3, 8-3
Pinus muricata (Bishop pine)..8-13, 5-8
Pinus remorata (Santa Cruz Island Pine)..5-8
Pinus contorta (Beach pine)..8-13
Pinus Murrayana (Lodgepole pine, Tamarack pine)..8-5,13-5,13-8
Pinus Torreyana (Torrey pine)..8-5,13-5
Pinus ponderosa (Yellow pine)..13-8, 13-5, 8-5
Pinus Jeffreyi (Jeffrey pine)..13-5, 13-8, 5-8
Pinus radiata (Monterey pine)..13-8, 8-5, 13-5
Pinus attenuata (Knobcone pine)..8-5, 13-5, 3-5, 3-8
Pinus Sabiniana (Digger pine).. 13-8
Pinus Coulteri (Coulter pine)..13-8

The same type of Fibonacci pattern is found in the cones of other conifers such as: coast redwood, mountain redwood, Douglas fir, mountain hemlock, white fir, red fir, Sitka spruce, Colorado spruce, etc.

STATISTICAL REGULARITY IN PINE CONES

The question is often raised: How constant are these Fibonacci patterns in nature? For pine cones, the results are very close to 100% for a given pattern. This does not mean that the deviants do not have a Fibonacci pattern but that possibly they have some other pattern.

In an investigation of 4290 cones of lodgepole, Jeffrey, silver, yellow, one-needled pinyon, foxtail, knobcone, Monterey, bishop and beach pines, there were 74 exceptional cones out of the 4290 or 1.7%. If the foxtail pine is eliminated, the per cent drops to 1.0. Note also that knobcone, bishop and beach pine showed no exceptions for the samples considered.

What about the cones that did not have the regular pattern? The situation can be illustrated by the foxtail pine deviants which were the most numerous for any one species.

7L-4R.............7
6R-4L.............4
6L-4R.............5
10R-6L............2
Mixed.............9 (pattern could not be found..some cones damaged)
9R-6L.............1
7R-4L.............3
7R-5L.............1
10L-6R............2

Some of these are Lucas patterns: 7L-4R, 7R-4L. 6R-4L is a double Fibonacci pattern, the numbers being twice 3 and 2. So also are 6L-4R, 10R-6L, 10L-6R. 9R-6L is a triple Fibonacci pattern, the numbers being three times 3 and 2. 7R-5L is a rarity, the numbers being from a sequence other than Fibonacci or Lucas.

STATISTICAL CORRELATION IN BISHOP PINE

As noted above, the bishop pine cone shows remarkable constancy in its pattern. Hence arose the following investigation. Take four or five feet of a branch of bishop pine which has cones and side branches. Find the count on the main stem between successive nodes. Determine the count on each of the side branches and on the cones in the various clumps along the stem.

Let us first dispose of some negative results.

(1) It is not true that all branches on a given pine tree show the same pattern for their needles.
(2) Nor is it true that on any branch, the branchlets coming out of one node are all the same in pattern: all 5R-3L, for example.
(3) Neither is it true that the cones at one clump are all 8R-5L; some may be 8L-5R.

But on the other hand, speaking more positively:

(1) Patterns along the main stem remained constant for all of the 144 branches studied.
(2) In 80% of the cases the pattern on the branchlets along a stem agreed with the pattern on the main stem.
(3) There was a strong tendency for the cones in a given clump to be spiraled in the same direction.
(4) In about 70% of the cases, the spiral direction on the cone agreed with the spiral direction on the main branch.

FIBONACCI COUNTS ON CACTI

There are quite a number of cacti (Opuntia) which do show Fibonacci and Lucas patterns, but these as a rule do not have the remarkable constancy of the pine cone. Here are the results of counts made on the cane cactus.

6L-4R...... 4	8L-5R...... 3
6R-4L......36	5R-3L......15
7L-4R...... 6	5L-3R...... 6
7R-4L...... 5	5R-4L...... 2
8R-5L......23	7R-5L...... 1

In addition to the regular Fibonacci patterns 8R-5L, 8L-5R, 5R-3L, 5L-3R, note the high incidence of double Fibonacci patterns 6L-4R and 6R-4L. In addition, there are Lucas patterns 7L-4R and 7R-4L as well as variant patterns 5R-4L and 7R-5L.

THE PINEAPPLE

The pineapple has always been cited as the classical example of Fibonacci numbers in nature. One can note as many as four spirals going through one particular eye of the fruit. For example, in one specimen, there were counts of 5-8, 5-13, 8-13. In a second, there were four different spirals which would provide six pairs. One particularly interesting case was a very gradual spiral which crossed a very steep spiral without having an eye in common with it and gave a count of 21-5.

SUNFLOWERS

The sunflower is a striking example of Fibonacci numbers in nature because the numbers involved are so large. Ordinarily, one observes one set of spirals going to the left and another to the right. The counts on these sets of spirals could be numbers such as: 55-89, 47-76, 34-55, 47-29,....One may also find a set of spirals directed more or less toward the center of the sunflower with a number of spirals the sum of the two given numbers. Thus for 55-89, this set of spirals would have 144 elements.

Brother Alfred Brousseau

SIXTH NATIONAL CONVENTION

The sixth national convention of Mu Alpha Theta is being planned for the first week of August, 1975, at Seattle University, Seattle, Washington, with support from Seattle Pacific College. If previous sessions provide a model, there will be a line up of fine speakers, student presentations, a math bowl contest, and tours with mathematics students and their sponsors attending from all over the nation.

Start planning now on money-making projects to send a delegation. More presentations by students or groups of students will be welcomed.

Some sponsors who are investigating transportation for students from their areas are:

Adele Hanson (Phone from Chicago-Milwaukee area)
8529 W. Chapman
Greenfield, WI 53228

Mary P. Truebger
820 Normal Cl.
Memphis, TN 38111

Paul Foerster
Alamo Heights I.S.D.
7101 Broadway
San Antonio, TX 78209

Bro. Leo Harvey
Rummel High School
P.O. Box 663
Metairie, LA 70004

Bea Young
Richardson High School
Richardson, TX 75080

Mr. E.N. Higgins, &
Mr. Robert Meyer
Tomahawk High School
Tomahawk, WI 54487
(From Chicago area)

Contact them if you are interested in group transportation from their area.

If other sponsors are interested in working on transportation from their area let the national office know, and we will publicize it in the next Log.

The general chairman of the convention is:
Mr. Robert Marion
Mount Si High School
Snoqualmie, WA 98063

He will be mailing you additional information later this fall.

PROBLEM SOLVING WORKSHOP

1. Suppose there are one million points inside a simple closed curve. Is there a straight line which separates the points into two sets, having 500,000 points in each, but does not pass through any of the points?

2. A man leaves to go up a mountain at 6 a.m. and arrives at the top at 6 p.m. The next day he leaves the top at 6 a.m., follows the same route down, and arrives at the bottom at 6 p.m. Is there a point on the mountain that he will reach at exactly the same time as the day before?

3. What is the maximum number of pieces possible if a pizza is cut 50 times?

4. What positive integer, if divided by 10 leaves a remainder of 9, divided by 9 leaves a remainder of 8, divided by 8 leaves a remainder of 7,........, divided by 2 leaves a remainder of 1? One answer is 14,622,042,959. Find a smaller solution. Is it the smallest?

5. What is the largest number which can be obtained as the product of positive integers which add up to 25?

6. Around a cylindrical tube, outside circumference 4", length 9", 10 turns of a wire are helically wound. The ends of the wire coincide with the ends of the same cylindrical element. Find the length of the wire.

7. A rectangular picture, each of whose dimensions is an integral number of inches, has an ordinary rectangular frame 1 inch wide. Find the dimensions of the picture if the area of the picture and the area of the frame are equal.

8. If a 4 x 5 rectangle is subdivided into unit squares, then a diagonal of the rectangle intersects the interior of 8 of these unit squares. How many unit square interiors does a diagonal pass through if the rectangle has dimensions 6 x 10? 8 x 12? 9 x 15? M x N?

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HYPERBOLIC GEOMETRY (continued)

are represented by Euclidean segments, whose endpoints are ideal points. Given any h-line l and an h-point P not on l, three types of h-lines can be generated. First, parallel h-lines r and q are determined by one of the ideal endpoints of l and the given h-point P. The second class of h-lines is composed of all those which intersect the given l, such as z. The third class comprises non-intersecting h-lines which share no real or ideal points in common with l, e.g. h-line y. The parallels r and q serve as limiting cases between the sets of intersecting and non-intersecting h-lines.

The perpendicular a, from P to l, forms two angles, α_1 and α_2, with parallels r and q. These angles are referred to as the angles of parallelism. It can be proven they are always congruent in the hyperbolic plane. An unusual property of the h-plane is that distances and angles are related. As perpendicular a lengthens (that is, as P moves away from l), the angle of parallelism decreases. Conversely, as a decreases, α_1 and α_2 approach 90°. This is stated by the following equations:

$$\lim_{a \to 0} \alpha = 90^\circ \qquad \lim_{a \to +\infty} a = 0^\circ$$

The Klein model enables us to visualize the two parallels found in hyperbolic geometry. Hyperbolic lines look "straight", but angles and distances are distorted.

For another representation of a hyperbolic plane, we may modify the Klein model somewhat, to form the Poincaré model. Again, the h-plane lies entirely within a Euclidean circle, in this case, Ω (fig. 3). As in the Klein model, the circle Ω is composed of ideal points not in the h-plane. H-lines are arcs of circles, such that where they intersect Ω, the Euclidean tangents are perpendicular. This includes diameters of Ω, which may be considered arcs of infinitely large circles. Where any two h-lines, s and t, intersect, the angles are determined by their Euclidean tangents, s' and t', at the point of intersection.

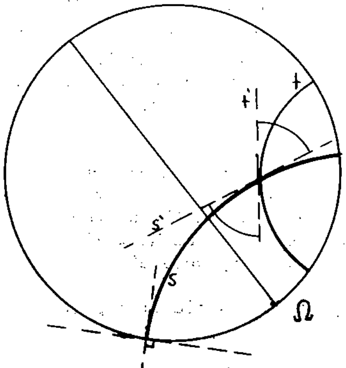

Fig. 3

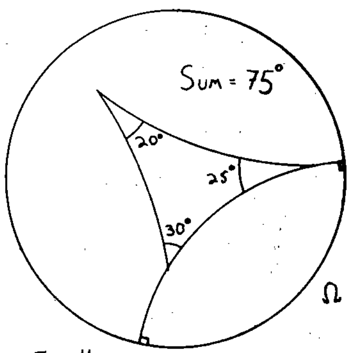

Fig. 4

The major advantage of the Poincaré model is its representation of angles in the h-plane. For instance, given a triangle formed by any three h-lines (fig. 4), inspection will show its angle sum to be less than 180°. Although only one simple triangle is shown, this can be proved for all hyperbolic triangles using the Sacheri Quadrilateral.

As regions are taken nearer and nearer the center of Ω, the h-lines falling within them approach the limiting cases of diameters, and therefore Euclidean lines. Therefore, near the center of Ω, the hyperbolic plane approximates a Euclidean one. Stated another way, Euclidean geometry merely approximates a small region in the hyperbolic.

Other interesting effects can be seen if one examines regions near the boundary. A theorem in hyperbolic geometry states that as the triangle angle sum approaches 0°, the area approaches a maximum. If one creates a triangle from any three near-ideal points (fig. 5, triangle A), the angle sum will be near 0°, and the area will be near its maximum limit. Using three ideal points, one obtains an "ideal" triangle whose area is a maximum (triangle B). Because both their areas are the same (the maximum), triangles A and B have equal areas.

Another theorem, which results from the interrelationship between distances and angles, states that similar triangles are congruent triangles. Indeed, because ideal triangles A and B both have three 0° angles, they are congruent! In case you haven't already guessed, distances are distorted in both the Poincaré and Klein models.

There have been a number of attempts to define a physical surface which would represent the hyperbolic plane. Any surface of negative curvature will approximate an h-plane, but one model in particular is generally used, which can be developed as follows:

Given a Cartesian plane with a toy railroad track along the positive x-axis with the engine at the origin, a weighted object is attached by a chain to the engine, and placed

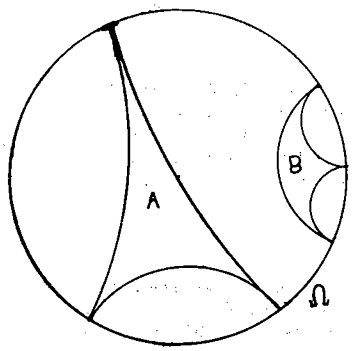

Fig. 5

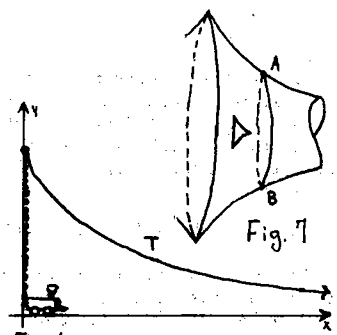

Fig. 6 Fig. 7

initially on the y-axis at some distance k from the origin (fig. 6). As the engine moves along the positive x-axis, the weighted object will describe curve T, asymptotic to the x-axis, called a tractrix. The tractrix is also described by the following equation, where k is the length of the chain:

$$x = k \cdot \log\left(\frac{k+\sqrt{k^2-y^2}}{y}\right) - \sqrt{k^2-y^2}$$

If the tractrix is revolved around its asymptote, the surface generated may be called pseudosphere. On a sphere the shortes path between two points determine geodesics which are arcs of great circles. On the surface of a pseudosphere we would like the geodesics to represent hyperbolic lines. However, any two points A and B which are exactly opposite each other lack a unique line (geodesic) connecting them (two can be drawn), which violates the first axiom. Nevertheless, the pseudosphere helps visualize many Lobachevskian properties, such as the triangle angle sum defect shown in figure 7.

To date, no completely satisfactory model of Lobachevskian geometry has been formulated.

Just as Newtonian physics proved to approximate Einsteinian physics, Euclidean geometry may only be an approximation of the real universe. Perhaps light waves travel in great circles in "spherical space" or along tractrices in hyperbolic space. Both are valid conjectures. The question "Which is the true geometry?" should be rephrased, "Which geometry simplifies the statement of the laws of physics?". This point of view is expressed by a dictum of Poincaré: "There is no true geometry. It is only a question of which is more convenient."

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