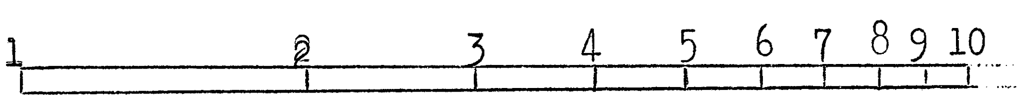
THE MATHEMATICAL LOG

of

THE NATIONAL HIGH SCHOOL AND JUNIOR COLLEGE MATHEMATICS CLUB

MU ALPHA THETA



VOL. I, no.2 Jan. 1958

Diophantine Equations

Many riddles and puzzles, both old and new, lead to equations of the form $ax + by = c$, where a,b,c are given integers, and x and y, the required unknowns, are also to be integers. The following example will serve as an illustration.

A man buys cows and pigs for a total of $1784. One cow costs $27 and a pig $14. How many cows and pigs can he buy?

Let x be the number of cows and let y be the number of pigs, then the equation

$$(1) \qquad 27x + 14y = 1784$$

has to be solved in positive <u>integers</u> x and y. One way to do this is to select the term with the smaller coefficient and solve for the corresponding unknown. In this case we solve (1) for y since 14 is less than 27, getting

$$y = \frac{1784 - 27x}{14} = \frac{(14 \cdot 127 + 6) - (2 \cdot 14x - x)}{14},$$

or

$$(2) \qquad y = 127 - 2x + \frac{6 + x}{14} = 127 - 2x + t \;,$$

where

$$(3) \qquad t = \frac{6 + x}{14} \;.$$

Since x and y in (2) are integers, t must be an integer. Conversely, solving (3) for x we see that

$$(4) \qquad x = 14\,t - 6$$

will be an integer if t is an integer. Substituting (4) into (2) we get

$$y = 139 - 27t.$$

We have thus obtained the solution

$$x = 14t - 6$$

$$y = 139 - 27t$$

of (1) in terms of an arbitrary integer t. For positive solutions we must take $t \geq 1$ to make x positive, and $t \leq 5$ to make y positive. Hence there are five possible values of t, that is t = 1,2,3,4,5 leading to the solutions:

$$x = 8 \quad , \quad 22 \quad , \quad 36 \quad , \quad 50 \quad , \quad 64$$

$$y = 112 \quad , \quad 85 \quad , \quad 58 \quad , \quad 31 \quad , \quad 4$$

For many problems of this kind as well as interesting historical comments, see Oystein Ore: Number Theory And Its History, McGraw-Hill Book Company, 1948, Chapters 6,7,8.

C. D. Olds
Dept. of Mathematics
San Jose State College

Meeting of The National Council of Mu Alpha Theta

The National Council of Mu Alpha Theta met in Kansas City, Missouri, November 30, 1957.

Present: President, Henry Alder; Vice-President, Edward Walters; Secretary-Treasurer, Mrs. R. V. Andree; Governors: Mr. George Hunt, Miss Nellie Kitchens, Miss Virginia Pratt.

Summary of the 7 hour session:

Governor Hunt made a motion that Mu Alpha Theta petition the M.A.A. for affiliation with that organization; NCTM be invited to nominate at least two members of the governing council. MOTION PASSED UNANIMOUSLY.

THE MATHEMATICAL LOG. After a discussion of the prices of mimeographing, lithoprinting and printing, Governor Pratt proposed that we mimeograph all three issues this year. EVERYONE AGREED. Items to be included were discussed--bulletin boards, reference books, scholarships, topology, calculus, number-theory, careers. Motion that we have a problem section in the MATHEMATICAL LOG and Dr. Hoggatt be asked to edit it. MOTION

CARRIED. Motion that Program Committee be called **Editorial** Committee. MOTION CARRIED. They will clear all articles on mathematics for publication in the LOG. Governor Kitchens suggested Secretary Andree continue to handle all news items. President Alder recommended that we send each chapter 3 or 4 copies of The Mathematical Log.

DISCUSSION OF PERIODICALS: Each chapter is now getting the Oklahoma University Mathematics Letter with the compliments of The University of Oklahoma. Governor Hunt recommended Mathematics Student Journal at 20 cents a year.

DISCUSSION OF WAYS TO MAKE GOOD REFERENCE BOOKS AVAILABLE TO OUR CHAPTERS: Lack of funds is the main problem. Governor Hunt suggested writing an article on a planned traveling library in the Mathematical Log.

Discussion of opportunities in mathematics.

Discussion of possible changes in the constitution: It was generally agreed that not all officers should be replaced at the same time. The eligibility of Junior College members was clarified.

Governor Kitchens moved that the council approve participation in the pilot program for visiting lecturers in high school, if and when set up by MAA. MOTION APPROVED. President Alder will write to them.

Secretary-Treasurer Andree read the treasurer's report. It was accepted. Mu Alpha Theta had $763 on hand November 1.

The meeting was adjourned.

Respectfully submitted,
Josephine Andree
(Mrs.) Josephine P. Andree
Secretary-Treasurer

GOOD READING.... possibly a meeting topic: "Paper Folding for the Mathematics Class" from the National Council of Teachers of Mathematics, 1201 16th St NW, Washington, D.C., 75¢.

* * *

SCHOLARSHIPS

This is an exciting era to be planning a college future in mathematics and science. The present offers wonderful openings for mathematicians and the future looks even brighter. Each of you is potentially a scientist with a splendid career, performing a real sefvice.

Don't let lack of money keep you from planning on college. Go after a scholarship! The U.S. Office of Education reported that 237,000 scholarships having a monetary value of $65.7 million were available to undergraduates in colleges in the school year 1955-56. Soon Congress will be considering a tremendous new system of scholarships. Probably any member of Mu Alpha Theta can qualify for some scholarship, somewhere. Your principal , guidance or class counselor may have abundant information on how to apply. If not, write to the colleges you consider attending, and ask them to send you information about their scholarships, as well as an application blank.

Do it now! Many colleges have to have your application on file in February. There is nothing to prevent your applying for several. Answer all the questions as completely as possible. For instance, if they ask what you plan to do with your training, give your present intentions even if you are not absolutely sure. After all, you are entitled to change your mind later if you find something else suits your talents better.

Good Housekeeping, November, 1957 has a list of scholarships. Copies of the publication " Financial Aid for College Students, Undergraduate" can be obtained from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., for $1.

Just keep remembering this! YOU CAN NOT GET A SCHOLARSHIP IF YOU DO NOT APPLY!!

YOUR BULLETIN BOARD

Maintaining an interesting bulletin board is a challenging and informative club project. It is best to have a standing committee in charge. All club members gather items and contribute ideas, but the committee is responsible to edit and display these contributions. Keeping the board attractive and up-to-date is vital. No display should remain over a month, and newsy clippings from periodicals should be changed weekly. Club members can, also, be kept informed about programs, dates, etc. through the bulletin board. It is well for the club to build its own board with the club name on it.

Examples of interesting items for the board are such things as the announcement that Japan is requiring more mathematics in her post war general education program, results of the accelerated program for college freshmen in mathematics at Golden, Colorado, recent articles by noted columnists about the need for mathematicians, and the recurring references to the Soviet training program for scientists and engineers.

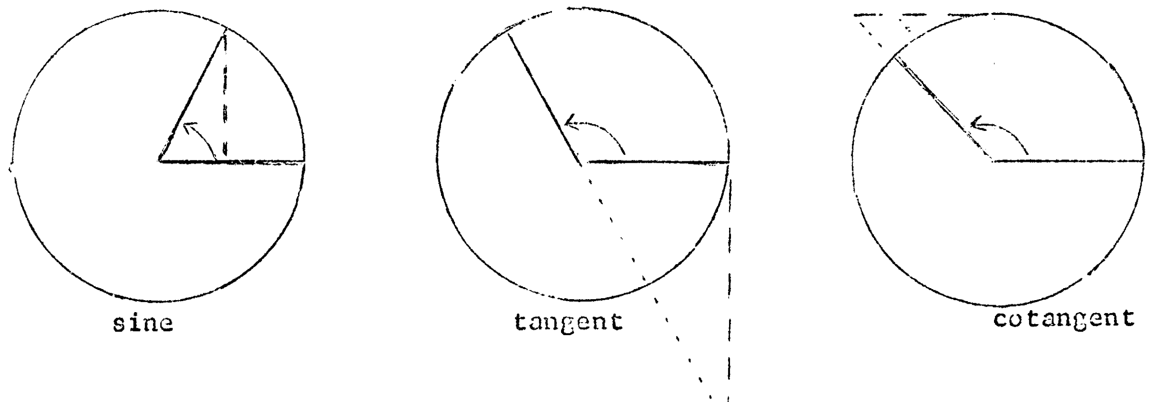
Job opportunities in the field of mathematics are, also, good bulletin board material. These can be had for the asking from the Armed Services, the U.S. Employment Service, and the U.S. Civil Service, and from many large oil companies. Scholarships offered by universities, colleges, and research institutes are, also, good.

Some good examples of displays that students may work out are the " Pictorial Representation of the Six Trigometric Functions", and the "Geometric Tree".

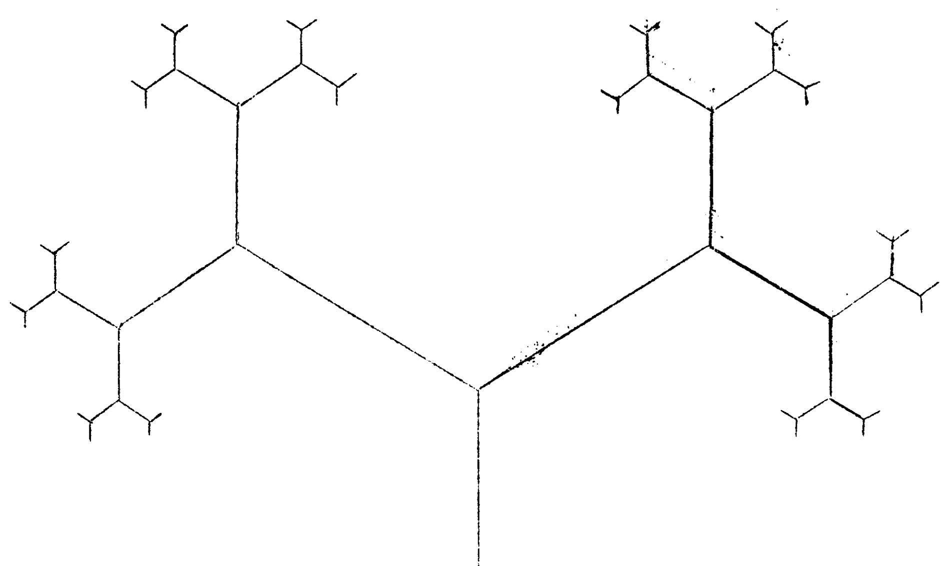
The first is practically self explanatory. The " Geometric Tree" is an attractive way to display the theory of approaching a mathematical limit. It is a good explanation of the way this theory is actually in operation in nature. Any mature tree is a similar approach to a limit. Many similar displays may be gleaned from class work on the appreciation of mathematics.

Challenging and intriguing problems also make good items. During rodeo season, one club offered the members of the rodeo club a prize for solving a puzzle about a mathematical steer. The problem was found in a comic book. The members ofthe club solved it correctly, checking with the answer in the next issu e of the magazine. The rodeo boys were unable to solve it.... they were a bit the best at riding the steer, though.

(display ideas- next page)



Samples of pictorial representation of the trigonometric functions. Use both acute and obtuse angles. The radius is one unit. The angle is in standard position. The curved arrow represents the angle in radians. The numerical value of the indicated function is represented by the dashed line- (red is more effective on a bulletin board.



The trunk of the tree is three inches long. This length was halved for the first two branches. This process is continued by halving the length of each branch for the two extensions on it. The branch and its two extensions are placed 120 degrees apart.

The process can be continued as long as mechanical precision will allow. The area occupied by the tree will not increase appreciably from its present size, no matter how many steps further the process is carried. This is an example of approaching a mathematical limit.

GEOMETRICAL CONSTRUCTIONS WITH LIMITED MEANS

Many Constructions require only a compass

by

C. D. Olds,
Department of Mathematics
San Jose State College
San Jose, California

Students learn in secondary school to carry out geometrical constructions with straight edge (ruler) and compass. There is hardly anyone, for instance, who could not bisect the circular arc AB with these two instruments. The procedure (fig. 1) is to swing two arcs of the same radius from centers A and B, and then to draw the line RS. This line will intersect the arc AB in its mid-point X. But suppose we did not have a straight edge with which to draw the line RS; how then could we find the point X? In other words, is it possible to bisect a circular arc with a compass alone?Here is the solution if AB is the arc of a circle with center at O(see fig. 2 and please get out a compass.) From A and B as centers, swing two arcs with radius AO. From O lay off arcs OP and OQ with radius equal to the distance AB. Then swing two arcs with PB and QA as radii and with P and Q as centers,intersecting atR.Finally, with OR as radius, describe an arc with either P or Q as center until it intersects AB at X, the required mid-point.

The beautiful construction just given was due to the Italian mathematician L. Masceroni (1750-1800), whose book LA GEOMETRIÁ DEL COMPASSO appeared in 1797. In this book he announced the very surprising theorem that all geometric constructions possible by straight edge and compass can be made by compass alone. Of course, one cannot draw the straight line joining two points without a straight edge, so that this fundamental construction is not covered by the Mascheroni theory. Instead, one must think of a straight line as being determined if we know any two points on it. For example, consider the simple problem of finding, with the aid of a compass alone, a point C on a line determined by two given points A and B and such that AB= BC. Clearly, all we need do is draw a circle about B of radius AB, and mark off on this circle , starting from A , the points P,Q,C, such that the chords AP=PQ= QC=AB. Thus without actually drawing lines, we are able to double a line segment AB (fig. 3)

Another interesting problem is that of finding the point midway between two given points, A and B, by using the compass alone. To solve this problem we have to introduce the notion of inversion of a point in a circle. The image of a point P with respect to a circle C is defined to be the point P' lying on the line OP on the same side of O as P and such that $OP \cdot OP' = r^2$, where r is the radius of the circle (fig 4). The points P and P' are said to be inverse points with respect to the circle C. Clearly, if P' is the inverse of P, then P is the inverse of P'. Given a point P, its inverse P' with respect to a circle may be constructed by means of a compass alone as follows (fig 5). With OP as a radius and P as center, we describe an arc intersecting circle C at the points R and S. With these two points as centers we describe arcs with the radius OR=r which intersect at O and at P'. Clearly P' is on the line OP. A little elementary geometry shows that $OP \cdot OP' = r^2$, so that P' is the required inverse of P.

It is now easy to find the mid point X of the line AB. Draw a circle of radius AB about B and use the previous construction to find the point X' such that $AB=BX'$. Draw the circle with radius AB and center A, and construct the point X inverse to X'with respect to this circle. Then $AX \cdot AX' = AX \cdot 2AB = AB^2$, or $2AX = AB$. Hence X is the desired mid-point.

Incidentally, if the reader will study the construction in Fig 5 carefully, he will be delighted to discover that he now is in possession of the solution to the problem of finding with compass alone the center of a circle whose circumference only is given.

In connection with Mascheroni's general theorem, it is enough for us to notice that any construction with ruler and compass permitted consists of a finite succession of the following elementary constructions, each of which is possible by means of the compass alone:

1. To draw a circle with given center and radius
2. To find the points of intersection of two circles.
3. To find the points of intersection of a straight line and a circle.
4. To find the points of intersection of two straight lines.

Using the compass alone it is clear that problems 1 and 2 are easy. But to solve 3 and 4 is much more difficult and the previous concept of inversion must be used. Problem 3 would be a real test of the reader's skill. It is, also, clear that once these four problems have been solved it is easy to invent problems at will, for all that is necessary is to select at random one of Euclid's propositions and see how it can be established by the use of the compass alone. Whatever the solution obtained, it is always interesting to turn to Mascheroni's book and see how the question is tackled there. His elegant solutions would be extremely hard to better.

Inspired by Mascheroni, the great geometer Jacob Steiner (1796-1863) singled out as a tool the straight edge instead of the compass. (by straight-edge, he meant an unmarked ruler, only one edge of which may be used, i.e., we are not permitted to use it to draw parallel lines.) The straight edge alone cannot suffice for all geometrical constructions in the classical sense, and the compass has to be used sometimes to help out; but nevertheless, Steiner was able to show that all constructions in the plane which are possible with straight edge and compass, are also possible with the straight edge alone, provided we are permitted to draw one fixed auxiliary circle and to mark its center. In general, the solution of problems of straight edge alone requires some knowledge of projective geometry. We will consider only a few simple constructions.

PROBLEM: (fig 7). On a straight line l three points P, Q ,R are given, such that Q is the mid-point of thesegment PR. Construct a parallel to l through a given point S not on l.

SOLUTION: Draw the lines RS and PS. From a point T on RS beyond S draw the lines TP and TQ. Then draw RM which intersects PT at N. The line NSwill be parallel to l. It is also evident that we now can solve the problem of finding the mid-point Q of a line segment PR provided a second line l' parallel to l is given.

Finally , suppose that two lines l and l'intersect at a point P outside the given sheet of paper (Fig 8). Without a compass construct a line through Q which would pass through P if it could be extended off the paper. To solve this problem draw in turn lines AA', AQB',A'QB and B'B. From D the intersection of A'A and B'B, draw in turn DCC', CB', and BC'. Let R be the intersection of B'C and C'B. The line QR is the one we set out to construct.

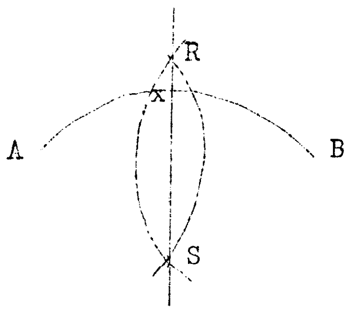


Figure 1

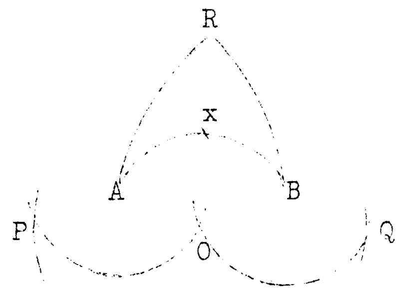


Figure 2

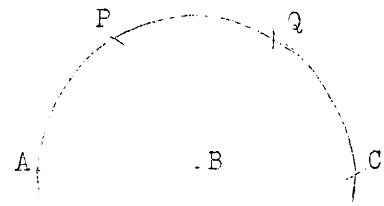


Figure 3

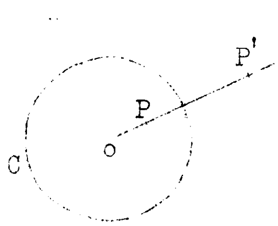


Figure 4

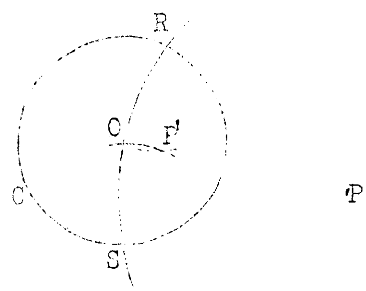


Figure 5

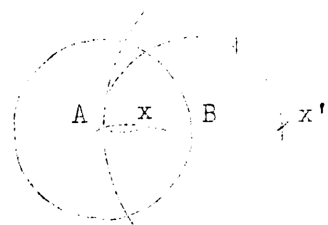


Figure 6

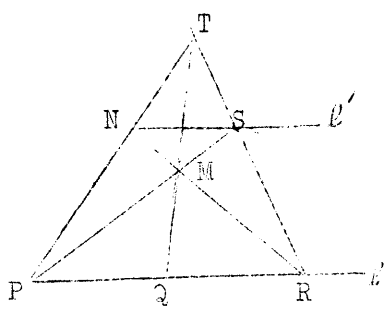


Figure 7

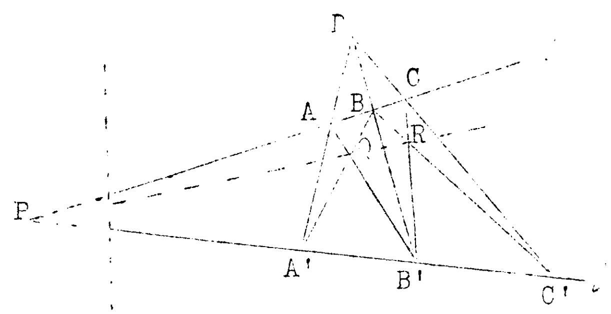


Figure 8

There is such a wealth of such fascinating problems awaiting the curious mind. For example, the reader will enjoy trying the following (using the straight edge only):

1. Double a given line segment, AB, when a parallel line 1 to AB is given.

2. Divide a segment AB into n parts if a parallel to AB is given.

3. If a fixed circle and its center are given, draw a parallel to a given straight line through a given point.

4. Given a fixed circle and its center, draw a perpendicular to a given line through a given point.

5. Construct a line joining two given points whose distance apart is greater than the length of the straight edge used.

The preceding examples give only a small sample of constructions carried out with limited means. It is hoped that they will inspire some students to investigate the subject further.

REFERENCES

1. Archibold, R.C., Constructions With A Double-Edged Ruler, American Mathe-Matical Monthly, vol. 25(1918), pp. 358-360.

2. Courant and Robbins, What Is Mathematics?, Oxford, 1941, (Chapter 3).

3. Stark, M.E., Construction With Limited Means, American Mathematical Monthly, vol. 48. 1941, pp. 475-479.

4. Hudson, H.P., Ruler and Compass, (included along with other monographs in the Chelsea Publishing Company reprint of Hobson's Squaring The Circle, 1953.

SHOULDER PATCHES

Some of our chapters want embroidered shoulder patches for sweaters or jackets so everyone will know their accomplishment in mathematics and their membership in the National High School Mathematics Club... and plugging mathematics, also. After consulting several firms that make such emblems, the best offer seems to be from a Chicago firm. They will make Swiss embroidered emblems in gold and light blue on felt in either

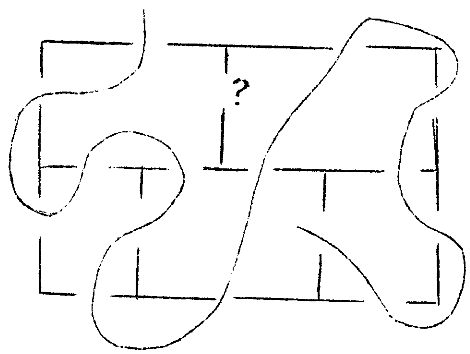
4 inch emblems with MU ALPHA THETA symbols as center motif, no lettering, at 55¢

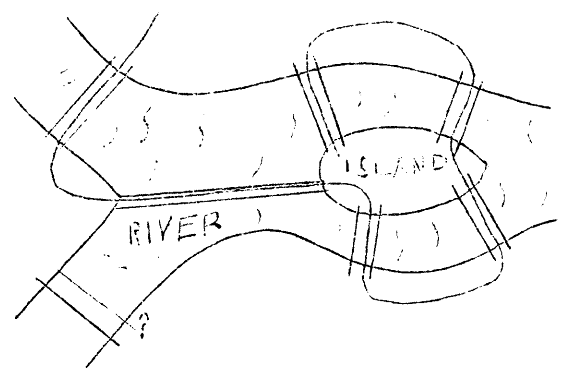
5 inch emblems with perimeter embroidered " National High School and Junior College Mathematics Club at $1.15

The smallest order is one loom of 170 emblems so the central office will have to place the order. Let us know if you are interested, and which style you prefer.The manufacturer is preparing a price on the Pythagorean triangle with only the word MATH below. Will each chapter vote at your earliest meeting and advise the national office your decision on this suggestion. We believe that the patch or letter with the word MATH will be about one-half way between the 55¢ and the $1.15. We will advise you the vote of the chapters and then, if the suggestion passes, orders can be placed with the national office.

TWO TOPOLOGICAL PUZZLES

Nearly everyone at one time or another has tried to solve the puzzle (fig 1) which consists of drawing a continuous curve which passes just once





through each " door" and " window" of the "house" without omitting any. In (fig 1), a " door:" has been omitted. In (fig 2), we have a similar problem, the famous Koenigsberg bridge problem. The question is whether it is possible to take a walk and cross each bridge once and only once and return to the starting point.

Neither of these puzzles can be solved and to explain why not makes a very interesting club topic. The Koenigsberg bridge problem was explained by Leonhard Euler (pronounced "oiler")in 1736. His original memoir has been translated and appeared in the July 1953 issue of the Scientific American. There is, also, a very readable explanation of this and similar problems in Chapter 9 of W.W.R. Ball's book, Mathematical Recreations and Essays, Macmillan, 1939.

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Department of Mathematics,
San Jose State College,
San Jose, California

GOOD READING

You will find an article in the American Mathematical Monthly, 1957, p. 557-565, on The Practice of Mathematics by R. E. Gaskell of the Boeing Aircraft Company, Wichita, Kansas... and it makes very good reading. If your teacher does not have a copyof this journal, she may obtain an application blank from your national secretary-treasurer, or from the University of your own state.

CAN YOU TOP THIS?

How many books has your school ordered from the National Science Foundation list sent out in October? Mr. David Dougherty has put the town of Ennis, Montana, on the map by ordering all of them! He writes, "I ordered all four sets of books selected for a high school math library; it almost seems like Christmas, what with getting a book or two every day. My superintendent and myself felt that in the long run the books would more than pay for themselves if only a few students use them every year. Besides we spend that much just on basketballs per year, at $20.00 a piece."

Surely you, too, can convince your principal or librarian that modern, interesting math reference books are a good investment.

PROPOSED TRAVELING LIBRARY

Have you a set of interesting mathematical reference books? No doubt you have persuaded your school library to add some new ones. Meanwhile maybe you would like the use of a few extra books for outside reading. Your National Council is making tentative plans for a traveling library for Mu Alpha Theta. As presently visualized, each library kit would contain 5 or 6 modern books of the kind in the National Science Foundation list you received last fall (not textbooks). A participating chapter would use these books about a month, then mail them to the next chapter on the list. Depending on how many chapters want to use the traveling libraries, you might get a different set of books the following month, or only on alternate months. Because there is a special postage rate on books, it would cost less than 50¢ to send each library kit along. There would be no other expense to you.

If you would like to use such a traveling library, write soon to:

Mrs. Josephine Andree
Box 1127, University of Oklahoma
Norman, Okla.

What books would you like to have included?

The enthusiastic letters from chapters coast to coast makes it a pleasure to open the mail here at the office in Norman, Okla. I wish each of you could read every letter. The following excerpts show some of the varied activities our members engage in.

Josephine Andree,
Secretary-Treasurer.

" You may be interested to know my club is now preparing for their fifth annual Arithmetic Contest which is sponsored by the Amvets of town and administered by the Math Club. Five of the best arithmetic students from each of the five grade schools in town are chosen by their school to participate. On December 17th, they will report to the high school at a set time, and take a test prepared by five (very carefully chosen) students chosen from my club. The President of the Club presides over the examination, students proctor and carry out other duties connected with it. The "Big 5" (as we call them) mark the papers. This is definitely a student project but , of course, I supervise very carefully. "

Bertha Godfrey,
Memorial High School,
West New York, New Jersey.

" We plan to have a lecture series for our monthly meetings consisting of speakers from nearby colleges."

Loren Johnson,
Sec-Treas.,ΜΑΘ Club,
Fontana HS, Fontana, Cal.

" You asked where mathematics teachers can apply for a scholarship in mathematics offered by the Du Pont Company. Interested parties as far west as Iowa and Oklahoma can write to:

Director of Special Programs,
Case Institute of Technology,
10600 Euclid Avenue,
Cleveland 6, Ohio."

Rev. William Matyas,OSB,
Benedictine High School,
Cleveland, Ohio.

" Out local club is sponsoring a Science Fair to be held February 26, 1958 . The Derry Westinghouse Corporation will award a first prize of $100 and a suitable plaque; Latrobe Steel Corporation, a $50 second prize and Keystone Alloys Company $20 third prize, for the best speeches and demonstrations. The local mathematics club will award three prizes of $10, $7.50 and $5 for the b best exhibits. We hope the fair will be a success and that it will continue to be held each year."

Edna Mae Love,
Derry Area Joint HS,
Derry, Pennsylvania.

" At the close of the initiation ceremony, all MAΘ initiates will be presented , with their parents, who will pin them. They will all be enclosed within a large paper chain of proper colors."

Blanche Moon,
Bethany High School,
Bethany, Oklahoma

" A discussion of **two** as a base for a number system proved to be a very popular subject for club meeting. Back numbers of The Mathematics Student Journal, Constance Reid's book, Zero to Infinity and the booklet Understanding Numbers by Philip Jones served as source material for the discussion. For one meeting, out club is sponsoring the film Base and Place from the series Understanding Numbers. The film will be shown to the entire student body."

Sister Mary Constantia,
Ward High School,
Kansas City, Missouri

" Our club is progressing fine with the next meeting a dinner meeting and the subject NON-EUCLIDEAN GEOMETRY. Three members of the club are reporting on the Lieber book. If an outstanding success (you never know!), we plan to do a program for the loval television station."

Marjorie Tillotson,
Juneau High School,
Juneau, Alaska.

" You should see the very beautiful and complete mathematical magazine Papyrus published by Erasmus Hall High School, Brooklyn, New York. It is superior."

Josephine Andree,
Nat'l Sec-Treas.

" A few started pestering (the enjoyable variety) me about Calculus and were wondering if they could not get extra help after school. After a few feelers were put out, a special meeting of the club was called, and the result is an after-school course with ten students making a serious study of Principles of Mathematics by Allendoerfer and Oakley. The way things are progressing here, some of our students will have a richer high school experience because of their participation in the mathematics club."

Arthur Peters,
St Johns Memorial HS,
Olathe, Kansas.

General Dynamics Corporation, 445 Park Ave., New York City 22, NY, has begun distribution of a long playing record by Edward Teller on "The Size and Nature of the Universe" and " The Theory of Relativity" Included with the recording is a new " Map of the Heavens"prepared by the National Geographic Society, and a picture caption booklet, " The Atomic Revolution", published by the corporatio which explains the theory and peaceful uses of nuclear fission and fusion. Why not write to them and get on their mailing list? Ask for the above material!

PROBLEMS AND SOLUTIONS SECTION

1. Prove by mathematical induction that

$$2^{n+3} \leqslant (n+3)!\ , \quad \text{for} \quad m \geq 1.$$

Here n is a positive integer and $k! = 1 \cdot 2 \cdot 3 \cdots k$ is the product of the first k positive integers.

2. Given the digits 1,3,4,7,9

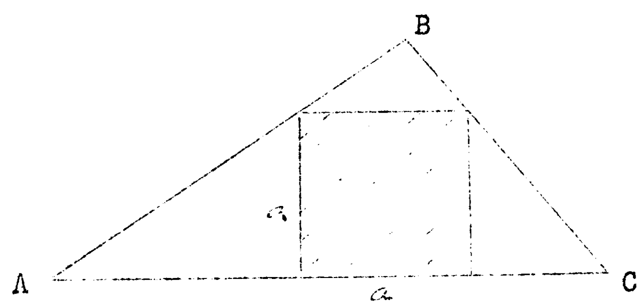
A. How many whole numbers can be formed using all of the digits if no number is to contain a repeated digit?

B. How many of the whole numbers will be even?

C. How many will be divisible by three?

D. How many will be dividible by four?

3. Show how to construct a square in the triangle shown:



4. A man bought some heavy electrical wire and the clerk mistook the number of yards for the number of inches and conversely. Not noticing this, the man used 9 9 inches and discovered he had twice as much left as he had originally ordered. If he had wanted an integralnumber of inches of wire, determine the shortest wire for which his order could havebeen placed.

5. A nitroglycerine charge with an impact fuze is dropped into an oil well casing of steel. The distance fallen in t seconds is given by $S = 16t^2$. The explosion is heard at the surface 9 seconds later. If sound travels 1100 feet per second in the air in the well casing, how deep is the well? If sound travels 16,000 feet /second in steel, when will this sound through the steel be heard. Next, suppose the exact time of release of charge is not known, but only that sound through metal precedes the sound through air by .3 seconds, how deep is the well?

problems cont'd

6. A man arising one morning finds himself in a pitch black room. Not wishing to awaken his wife, he did not turn on the lights. In the dresser are 16 white socks and 16 black socks mixed together. He decides to dress in the kitchen. How many socks must he take with him to get a pair of matched socks? How many socks must he take to get a pair of white socks? How many must he take to get a pair of each color? How many must he take to get some two pair?

7. Four nines may be used with high school mathematics symbols to represent all of the whole numbers from one through one hundred.

For example: $6 = \sqrt{9} + \sqrt{9} + 9 - 9$, $12 = (\sqrt{9})! + \sqrt{9} + 9/\sqrt{9}$
Find as many as possible representations of 64 using four nines (Solutions are desired by the editor).

8. If the elements of a third order determinant are the first nine positive digits, 1, 2, 3, 4, 5, 6, 7, 8, 9 used once and only once, many values for the determinant will occur. How many different values for this determinant can you find. (The editor desires carefully checked answers to this.)

9. If two points are too close together, the ruler can not be placed to draw the line, hence we will say such a line is not well defined. Find an indirect construction which will allow such a line to be drawn, and thus avoid the direct ill-defined construction.

V. E. Hoggatt,
Problems & Solutions Editor,
San Jose State College,
San Jose, California.

ALL TEACHERS ATTENTION!

The National Science Foundation is sponsoring a large number of summer institutes for high school teachers. YOU should go. NSF will pay you about $75 per week, plus $15 per week for each dependent.(maximum 4) plus transportation (maximum $80) to attend one of these institutes. They can't award a stipend to you IF YOU DON't apply, so apply, and apply now. Applications must be in the hands of 1the various institute directors by FEBRUARY 15, 1958. Write to the individual director for application blanks and apply to several institutes for admission. ACT NOW! A partial list of mathematics institutes follows: UNIV OF BUFFALO, Buffalo 14, NY, Harriet F. Montague, Department of Mathematics, July 7-August 1; CARLETON COLLEGE, Northfield, Minn., Kenneth O. May, Dept. of Math & Astronomy, June 16-July 25; UNIVERSITY OF CHICAGO, Chicago 37, Ills., Alfred L. P tnam, 411 Eckhart Hall, June 23-August 1; **UNIV. OF NOTRE DAME**, Notre Dame, Ind., Arnold E. Ross, Dept of Math, June 18-August 6; OBERLIN COLLEGE, Oberlin, Ohio, Wade Ellis, Dept of Math, June 16-August 8; RUTGERS UNIV., New Brunswick, N.J., Emory P. Starke, Dept of Math , June 29-Aug 8; UNIV OF VERMONT, Burlington, Vt., N James Schoonmaker, Dept of Math, June 23-August 8; WESTERN MICHIGAN UNIV.,Kalamazoo, Mich., Charles H. Butler, Dept of Math, June 23, August 8; UNIV OF WISCONSIN, write H. Van Engen, Dept of Math, Iowa State College, Iowa State, June 30-Aug 22; UNIV OF WYOMING, J. Norman Smith, D pt of Math, Laramie, June 16-August 8;UNIV OF KANSAS, Lawrence, G. Baley Price, Dept of Math, June 9-August 2.

ANSWERS TO PROBLEMS IN VOL. 1, MATHEMATICAL LOG

D&J	615
ANDREE	086322
SEND	7286
CHEER	94223

with $E^2 = H$.

Merely finding a set of digits which satisfy the problem is only the start. The real work comes in showing that the solution you found is unique, i.e., that no other set of digits have the desired addition property and also the property that $E^2 = H$.

John's 60 inch gun can be packed in a cubical box one yard on a side which meets baggage requirements and has a diagonal of $\sqrt{3}$ yards $\geqq 62.3$ inches

---

30 mph one way and 60 mph on the return trip gives what average speed? Let D represent the distance traveled one way. Then,

total time for the trip = time going + time returning

$$T = \frac{D}{30} + \frac{D}{60}$$

$$T = \frac{D}{20}$$

Thus the total distance, 2D, is covered in $\frac{D}{20}$ hours at an average speed of $\frac{2D}{D/20} = 40$ mph.

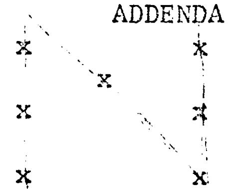
Our congratulatins are reserved, however, for those students who also obtained the general relationship

$$\frac{2}{X} = \frac{1}{V_1} + \frac{1}{V_2}$$

where X is the average speed for the entire trip, while V_1 and V_2 are the average speeds each way. Our special congratulations

(cont'd next page)

ADDENDA

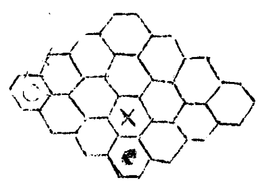


to those teachers who recognized that X is the harmonic mean of V_1 and V_2 and who pointed this out to their students. An article on the harmonic mean is contained in the Vol. V, No. 1 (September, 1955) issue of the O. U. Mathematics Letter, obtainable from Professor Dora McFarland, University of Oklahoma, Norman by sending 20 cents and requesting the specific issue.

If $U_0 = 1$, $U_n = 1 + 2U_{n-1}$, find U_{10}.

$U_{10} = 1023$, but again we reserve our congratulations for those students who discovered and proved the general rule

$U_n = 2^n - 1$.



$4(102564) = 410256$. Again it may be possible to discover a general method which will enable you to work similar problems for any digit K.
